Unified Media Programming: An Algebraic Approach

Simon Archipoff, David Janin, LaBRI, Bordeaux INP, University of Bordeaux

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Plan of the talk

▶ Preamble: a glimpse of the future
▶ Opening: the turtle and its pen
▶ Theme I: monoid semantics
▶ Theme III: resettable monoid semantics
▶ Variation: the turtle and its time machine
▶ Finale: demo and conclusion

▶ More in the paper: Theme II: inverse monoid semantics
A glimpse of the future
Preamble

Programmer office

from *Zero Theorem* by Terry Gilliam, 2013.
Preamble
Programming device

from *Zero Theorem* by Terry Gilliam, 2013.
Preamble

Programming interface

from *Zero Theorem* by Terry Gilliam, 2013.
Preamble

Spinning the metaphor

The Terry Gilliam “correspondance”’

Program = proof = building

and

Programmer = prover = builder
The turtle and its pen
Opening: a turtle equipped with a pen
Opening: the pen can be moved on or off the screen
Opening: the pen can be moved on or off the screen
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Drawing a point when pen is on...
Opening: the turtle can walk
Opening: the turtle can walk
Opening: the turtle can walk
Opening: the turtle can walk

drawing segments when pen is on...
Opening: the turtle can walk
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drawing 2D figures (or more)…
Program syntax

A program is a sequence of elementary actions

- *Flip* pen on or off
- *Walk* $d$ for some distance $d$
- *Turn* $d$ for some angle $d$
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The square example

Walk 1, Turn $\pi/2$, Walk 1, Turn $\pi/2$, Walk 1, Turn $\pi/2$, Walk 1, Turn $\pi/2$!
Monoid semantics
Programs form a monoid. We aim at defining a monoid semantics model for turtle program.
Semantics elements: positions, figures, moves and drawings

State space

- Positions: \( P = \mathbb{R}^2 \times \mathbb{R}/2\pi \mathbb{R} \times \mathbb{B} \),
- Figures: \( F = \mathcal{P}(\mathbb{R}^2 \times \mathbb{R}^2) \).
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Moves and drawings

- Moves: \( M = P \rightarrow P \),
- Drawings: \( D = P \rightarrow F \).
Monoid semantics

Elementary action semantics

Move semantics: $P \rightarrow P$

- $\llbracket Flip \rrbracket_M(p, a, b) = (p, a, -d)$
- $\llbracket Walk \ d \rrbracket_M(p, a, b) = (p + (d \times \cos(a), d \times \sin(a)), a, d)$
- $\llbracket Turn \ d \rrbracket_M(p, a, b) = (p, a + d, b)$
Monoid semantics

Elementary action semantics

Move semantics: \( P \rightarrow P \)

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\( \left[ \text{Turn } d \right]_M(p, a, b) = (p, a + d, b) \)

Drawing semantics: \( P \rightarrow F \)

\( \left[ \text{Flip} \right]_D(p, a, b) = \begin{cases} 
\{(p, p)\} & \text{when } b = 0 \\
\emptyset & \text{when } b = 1 
\end{cases} \)
\( \left[ \text{Walk } d \right]_D(p, a, b) = \begin{cases} 
\{(p, p + (d \times \cos(a), d \times \sin(a)))\} & \text{when } b = 1 \\
\emptyset & \text{when } b = 0 
\end{cases} \)
\( \left[ \text{Turn } d \right]_D(p, a, b) = \emptyset \)
Monoid semantics

The monoid

\[ S = \underbrace{P \rightarrow P}_M \times \underbrace{P \rightarrow \mathcal{P}(F)}_D \text{ equipped with product} \]

\[ (m_1, d_1) \cdot (m_2, d_2) = (m_2 \circ m_1, d_1 \cup d_2 \circ m_1). \]
Monoid semantics

The monoid
Set $S = \underbrace{P \rightarrow P}_{M} \times \underbrace{P \rightarrow \mathcal{P}(F)}_{D}$ equipped with product

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Program semantics
Inductively define by composition of elementary action semantics

- $[\epsilon] = \emptyset$ with empty program $\epsilon$,
- $[ap] = ([a]_M, [a]_D) \cdot [p]$ for all elementary action $a$ and program $p$. 
Monoid semantics

The monoid

Set $S = \prod \mathcal{M} \times \prod \mathcal{D}$ equipped with product

$$(m_1, d_1) \cdot (m_2, d_2) = (m_2 \circ m_1, d_1 \cup d_2 \circ m_1).$$

Program semantics

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- $\left[ \epsilon \right] = \emptyset$ with empty program $\epsilon$,
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Remark

Semantics is the morphism generated from the (free) monoids of programs into the semantics monoids.
Monoid semantics

Lemma

- moves under (flipped) composition form a monoid,
- drawings under (element-wise) union form a monoid,
- moves act by endomorphisms over drawings by \( m \star d = d \circ m \),
- and we have \( S = M \ltimes D \), i.e. semantics monoid is a semi-direct product.
Resettable monoid semantics
Resettable monoid semantics

Program reset

For every program $p$, define $\text{reset}(p)$ by $\llbracket \text{reset}(p) \rrbracket = (id, \llbracket p \rrbracket_D)$. 
Resettable monoid semantics

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General construct
Given a monoid $M$ acting by endomorphisms on a lattice $L$, the semi-direct product $M \ltimes L$ is a resettable monoid.
Resettable monoid semantics

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Given a monoid \( M \) acting by endomorphisms on a lattice \( L \), the semi-direct product \( M \ltimes L \) is a resettable monoid. That is, given \( (m, d)^R = (1, d) \), we have:

- \( (M \ltimes L)^R = \{1\} \ltimes L \) is a idempotent commutative submonoid,
- \( (m, d)^R \) is the least left unit of \( (m, d) \).
Resettable monoid semantics

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This is known in the York school as a \textit{left semi-adequate} monoid.
Resettable monoid semantics

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Moreover, for all $x, y, z \in M \ltimes L$, we have:
- $x^R = y^R \Rightarrow (zx)^R = (zy)^R$ (left Ehresmann),
- $(xy)^R x = xy^R$ (left restriction).
Resettable monoid semantics

Extending Turtle Programs

- Moves can be extended to non-injective moves, e.g. projection.
Resettable monoid semantics

Extending Turtle Programs

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Interpretation of the reset

- Resets act as kind of a fork operator: $\text{reset}(p) <> q$ can be understood as “fork a sub-turtle behaving like $p$” and “keep on executing $q$”.
The turtle and its violin
The violin metaphor

We want our turtle “to play violin” or, more seriously, to act also over the time dimension...
Temporal turtle semantics

Temporal moves and animations

Let $T$ be a timescale, e.g. $T = \mathbb{R}$.

- Temporal moves: $TM = T \rightarrow T$ (possibly partial),
- Animation $A = T \rightarrow S$,

where $S = (M, D)$ is the (inverse monoid of) turtle 2D semantics.
Temporal turtle semantics

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Temporal semantics
- $TM$ is a monoid under flipped composition,
- $A$ is a monoid under point-wise extension of the 2D product,
- $TM$ acts by endomorphism on $A$ by $tm \ast a = a \circ tm$
  (with $a \circ tm(t) = \emptyset$ in the case $tm(t)$ is undefined),
so that $TS = TM \ltimes A$ is a inverse (or at least resettable) monoid
for temporal turtle program semantics.
Temporal turtle elementary programs

Examples

Bijection over time:

- $\mathbb{D}[\text{Delay } d] = (t \mapsto t + d, \epsilon),$
- $\mathbb{D}[\text{Stretch } f] = (t \mapsto t \cdot f, \epsilon),$

or partial bijection over time (extending action of TM over $A$ accordingly)

- $\mathbb{D}[\text{Start}] = (t \mapsto t \text{ if } t \geq 0, \epsilon),$
- $\mathbb{D}[\text{Stop}] = (t \mapsto t \text{ if } t \leq 0, \epsilon),$

which can be combined with delay to define $\text{Play } t_1 \text{ } t_2$ that cuts the timescale from $t_1$ to $t_2...$
Temporal turtle elementary programs

Examples

Bijection over time:

- \([ \text{Delay } d ] = (t \mapsto t + d, \epsilon)\),
- \([ \text{Stretch } f ] = (t \mapsto t \times f, \epsilon)\),

or partial bijection over time (extending action of TM over \(A\) accordingly)

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- \([ \text{Stop} ] = (t \mapsto t \text{ if } t \leq 0, \epsilon)\),

which can be combined with delay to define \(\text{Play } t_1 \ t_2\) that cuts the timescale from \(t_1\) to \(t_2\)...

A programming API

In a modern language such as Haskell, the above functions may be part of a \textit{timed monoid} class type.
Musical extension

Let $E$ be a set of musical events, e.g. note on $n^+$ and off $n^-$ one for each note $n \in N$. Let $T$ be the symbolic temporal scale.
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Symbolic music monoid

Consider $D = T \rightarrow T$ (partial) and $P = T \rightarrow \mathcal{P}(E \times T)$ with

$$M = D \rtimes P$$

and the induced (resettable) submonoid generated by

$$\llbracket n \rrbracket = (t \mapsto t + 1, t \mapsto \{(n^+, t), (n^-, t + 1)\})$$

one for each note $n \in N$ together with Delay, Shift, Start and Stop.
Musical extension

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and the induced (resettable) submonoid generated by

$$[\lfloor n \rfloor] = (t \mapsto t + 1, t \mapsto \{(n^+, t), (n^-, t + 1)\})$$

one for each note $n \in N$ together with Delay, Shift, Start and Stop. Then, the monoids $M$ contains all finite polyphonic symbolic melodies.
Our actual implementation
3D by extrusion
Painting vs extruding

Extrusion is easier

- pen down: “control” points (or curves),
- move: combining complex moves,
- pen down: “control” points (or curves),
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- etc..

and compute automatically... curves (surfaces), curve tangents (or surface normals), resolutions, interpolations...
Painting vs extruding

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and compute *automatically*... curves (surfaces), curve tangents (or surface normals), resolutions, interpolations...

**Drawing specification**

With reset, a drawing specification is a tree structures set of “control points”...
From Haskell to GPU

A three layer architecture

- Haskell animated 3D scene syntax,
- Haskell animated 3D scene specification,
- GPU rendering.

Efficiency

- 5000 drawing elements specified (on CPU)
- up to 1000000 triangles drawn (on GPU)
- at decent animation rate (from 30 to 70 fps).
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Drawing examples
Drawing examples
Drawing examples
To do list
Normal forms

The need for normal forms
So far, we have seen ways of synthesizing/writing temporal media. Temporal media transformations require ways of analyzing/reading temporal media. Normal forms should allow this.

Stronger needs when on-the-fly
Temporal media must be read (in reactive program) in a time coherent way.
Other needs

► Generic handling of piles of semi-direct product (much like Monad transformers)
► Cut monoids vs enveloppe monoids (dual construction ?)
► Richer geometry (with bounded size instances to be sent to the GPU),
► Heterogeneous dimension (points, curves, surfaces, volumes, etc...),
► Examples library (pandemonium)
Octopus on the grid:

https://github.com/OctopusFarm/Octopus