Tiled Polymorphic Temporal Media

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FARM'14

Gothenburg, Sweden
September 6, 2014

Motivation

Our goal is to design a system for combining multimedia objects (such as sound, video, and animation) that is:

- Simple (a beginner can use it)
- Efficient (runs fast, efficient transformations)
- Elegant (concise, perspicuous)
- Sound (strong mathematical properties)

One Approach:
Polymorphic Temporal Media

- *Polymorphic Temporal Media* (PTM) [Hudak '04,'08] has four key elements:
  - A *neutral* value (e.g. silence or transparency)
  - A set of *primitive* values (e.g. notes or video frames)
  - A *binary sequential composition operator* :+: such that p1 :+: p2 is "p1 followed by p2."
  - A *binary parallel composition operator* :=: such that p1 :=: p2 is "p1 in parallel with p2."
- PTM is the basis of Haskore [Hudak '96] and Euterpea [Hudak '13]

PTM Properties

- Nice algebraic properties:
  - (p1 :+: p2) :+: p3 == p1 :+: (p2 :+: p3)
  - (p1 :=: p2) :=: p3 == p1 :=: (p2 :=: p3)
  - p1 :=: p2 == p2 :=: p1
  - rest 0 :+: p == p == p :+: rest 0
  - if dur p1 = dur p3 then
    (p1 :+: p2) :=: (p3 :+: p4) ==
    (p1 :=: p3) :+: (p2 :=: p4)
- Indeed, there exists a set of axioms that is *sound* and *complete* [Hudak '08].
- PTM is *polymorphic* (sound, music, video, animation, robot movement).
- PTM is also *efficient*, and arguably *simple* and *elegant*.

PTM Shortcomings

But all is not rosy… PTM has certain shortcomings:

- Semantics:
  p1 :=: p2 has several interpretations: p1 and p2 may start at the same time, or end at the same time, or be centered in time. None are inherently the right choice.
- Expressiveness:
  PTM doe not distinguish between “logical” and “actual” start and end times. For example:
  - In music, a measure may have a “pickup” (or *anacrusis*) – logically, it begins when the measure begins, but actually it begins when the pickup begins.
  - In video, a clip may have a “fade-in” – logically the beginning is the main clip, but actually it is the fade-in.

A Solution: *Tiled* PTM

- At FARM '13 David Janin presented the *T-Calculus* which, at least conceptually, eliminates the shortcomings of PTM.
- It can be viewed as *tiling* the one dimension of time.
- However, it has its own shortcomings:
  - It has no implementation.
  - It is constrained to the power of finite state transducers.
  - Its connection to inverse semi-group theory is interesting, but weaker than it could be.
  - Its recursion scheme (for infinite terms) is weak.
- Our solution:

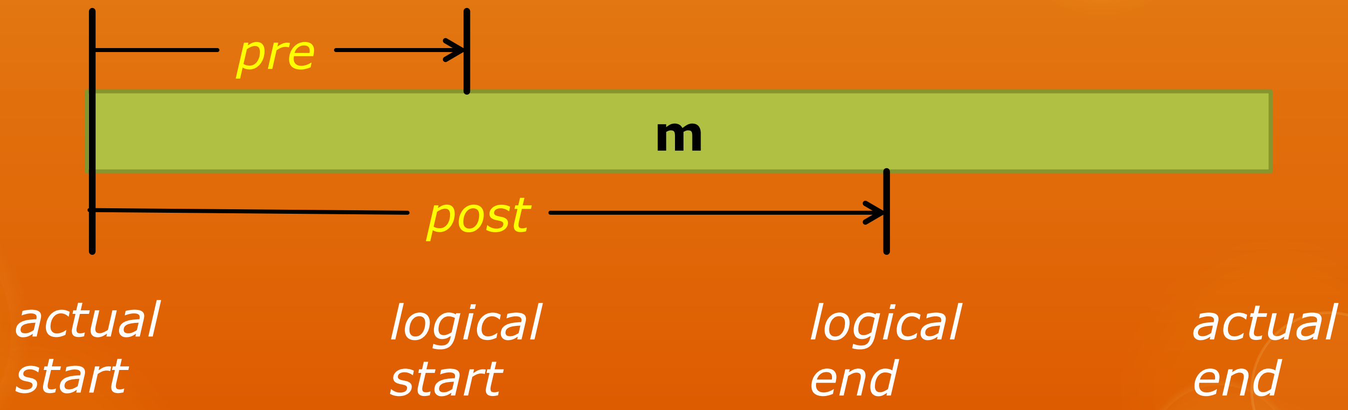
T-Calculus + PTM = *Tiled PTM*

Contributions

- The design of ***Tiled PTM*** (T-PTM)
  - Combines best attributes of PTM and the T-Calculus
- Discovery of new ***algebraic properties***
  - In particular, a stronger connection to inverse semi-groups
- Exploration of effective ***recursion schemes***
  - Allows infinite tilings
- An ***implementation*** in Haskell
  - Specifically, in Euterpea (i.e. PTM constrained to music)

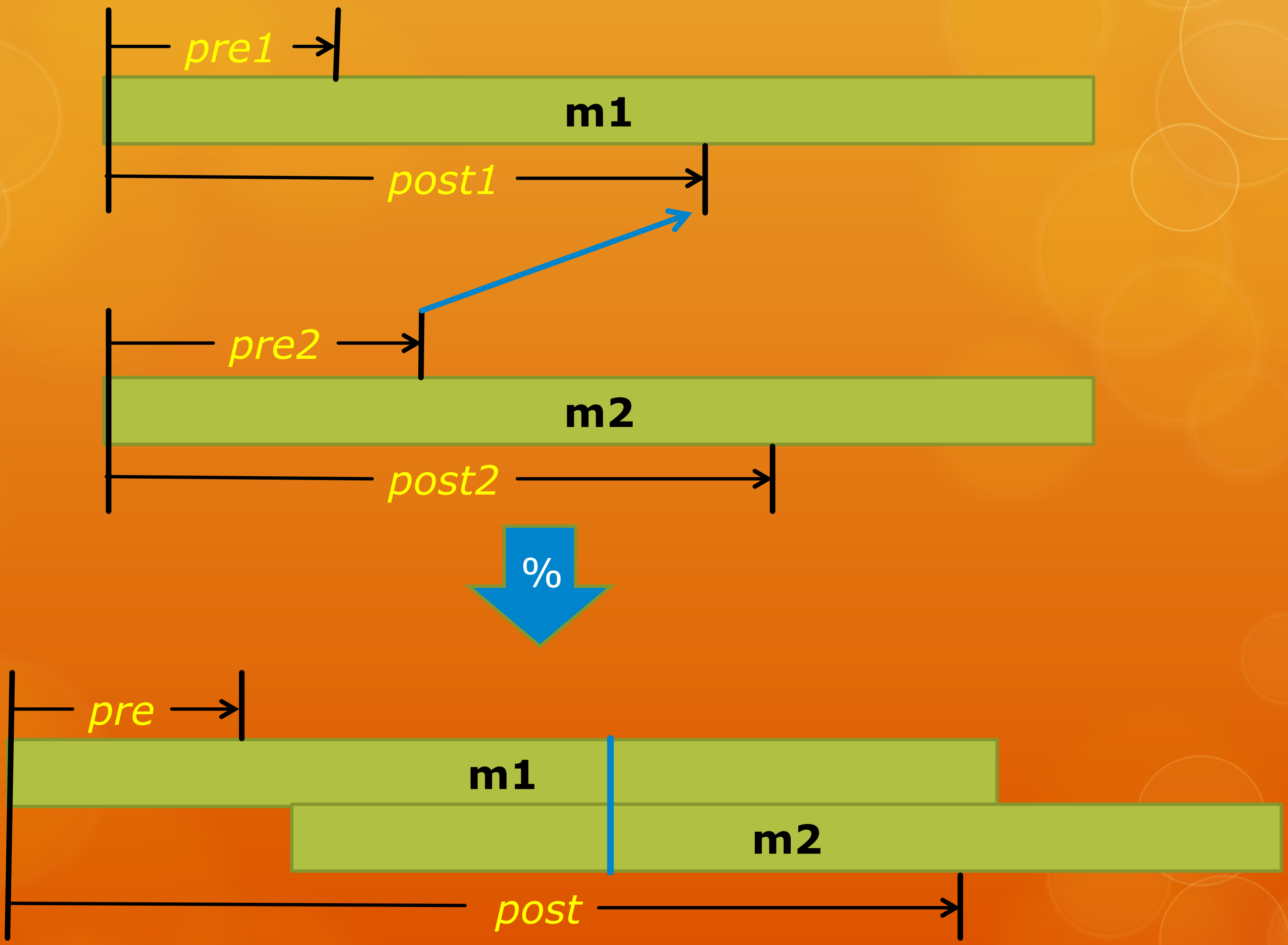
Basic Idea

- A tiled PTM has two *synchronization marks:*
  - *pre* marks the *logical start*, relative to the actual start
  - *post* marks the *logical end*, also relative to the actual start
- Pictorially:



Tiled Product

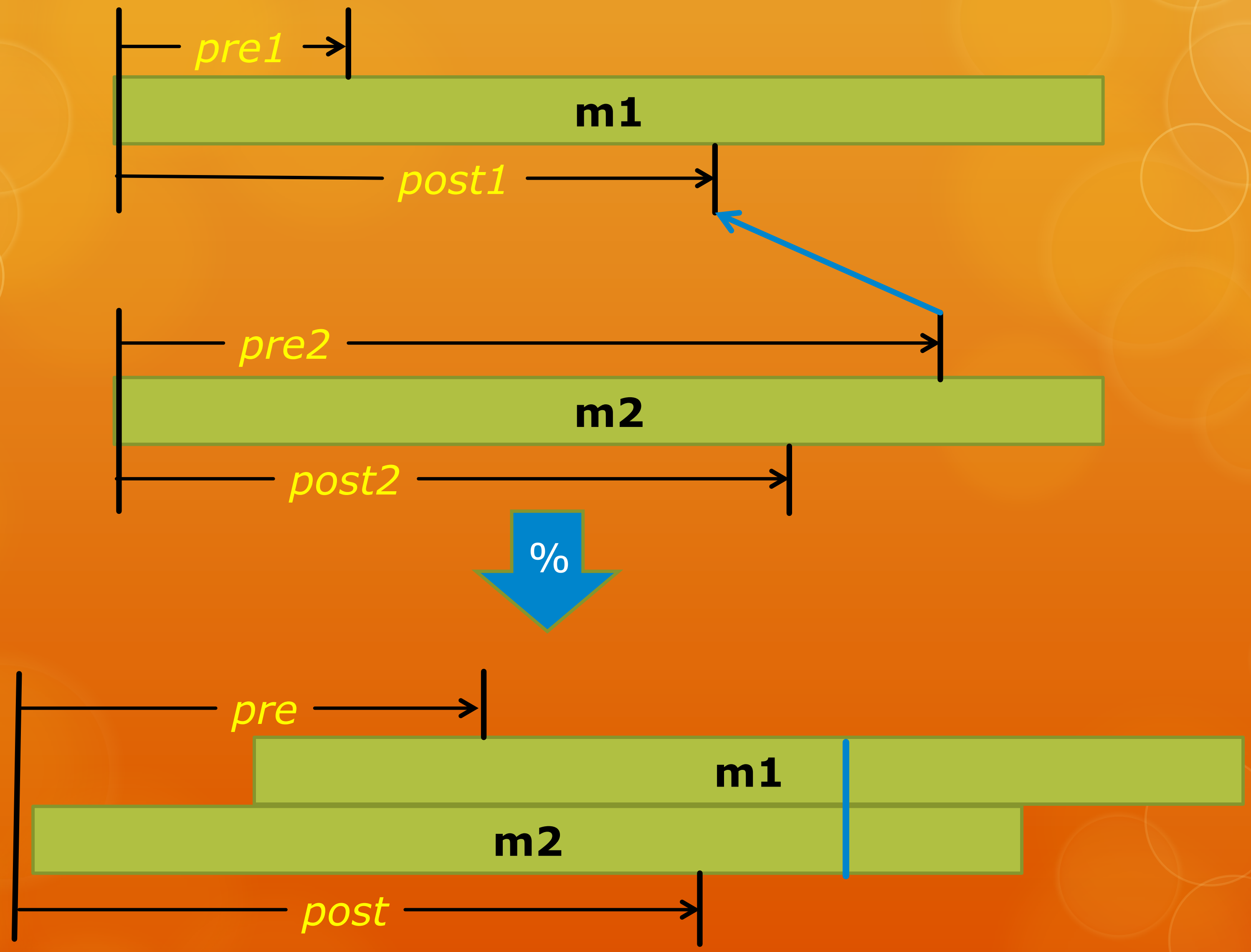
- Two tiled PTM values can be combined by a binary *tiled product* operator %.
- m1 % m2 is a tiled PTM that is the tiled product of m1 and m2.
- This involves:
  - *Synchronization* of the logical start of m2 with the logical end of m1.
  - *Fusion* of the overlapping content of m1 and m2.
- Pictorially: [next slide]


pre1
m1
post1
pre2
m2
post2
%
pre
m1
m2
post

Some Key Points

- In the construction of a tiled product, *partial overlap* may occur.
- So it is neither a sequential product nor a parallel product – *it is both.*
- :+: and :=: can be encoded in terms of %.

- In a given tile, *pre may be greater than post*!
  - In which case, what is the meaning of tiled product?


pre1
m1
post1
pre2
m2
post2
%
pre
m1
m2
post

Algebraic Prperties

- With a suitable notion of *observational equivalence* (see paper) we can show:

  (t1 % t2) % t3 == t1 % (t2 % t3)

- T-PTM's *neutral* (or *silent*) tile r d has duration d, and:

  r 0 % t == t == t % r 0

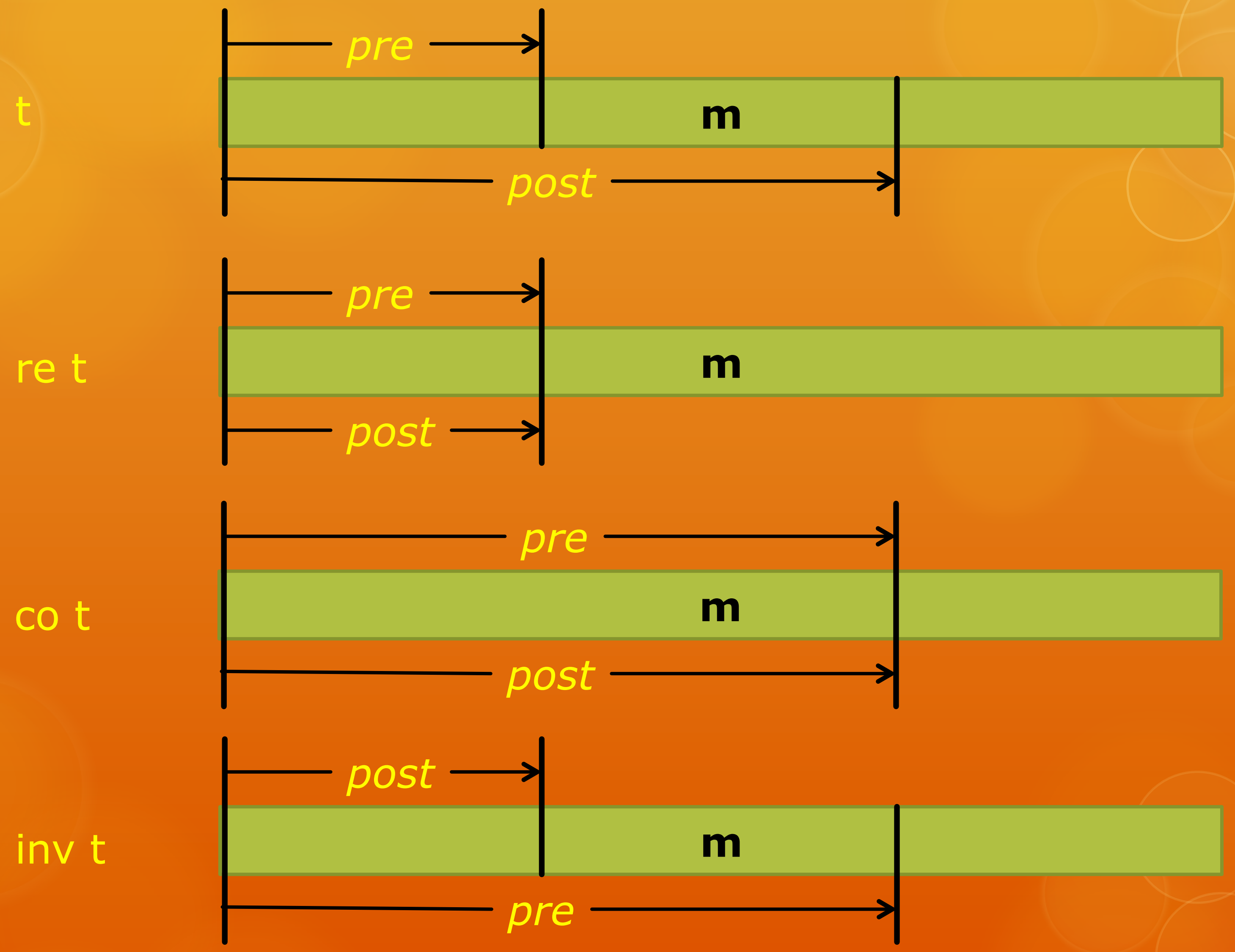
- Therefore T-PTM is a *monoid.*

Other Operators

- *Primitive monomorphic values*:
  - In music, t n o d is a musical note with pitch class n, octave o, and duration d. E.g. t C 5 (1/4) is middle C with quarter note duration.

- Reset re
- Co-reset co
- Inverse inv

[let's look at pictorial descriptions]


t
pre
m
post
re t
pre
m
post
co t
pre
m
post
inv t
post
m
pre

Example of Modularity

- let m1 = pm :+: r d0 :+: pu
  in m1 :+: m2
- Now suppose we lengthen pu:
  let m1 = pm :+: r d1 :+: pu
  in m1 :+: m2
- Suppose pu becomes sufficiently large:
  let m1a = pm
  m1b = r d3 :+: pu
  in (m1a :=: m1b) :+: m2
  - More modular, but still lacks logical structure
- In contrast, with T-PTM:
  m1 % (co pu % m2)
  - Fully modular: *changes to* pu *induce no changes to* m1 *or* m2*.*
  - Has logical structure

Why inv? Why "negative" tiles?

- Reset and co-reset can be defined in terms of inv:
  - re t = t % inv t
  - co t = inv t % t
- In an *inverse semi-group* the *inverse* of an element x is an element y such that:

  x . y . x = x   and   y . x . y = y

  Now note that in T-PTM:

  t % inv t % t = t   and   inv t % t % inv t = inv t
- Therefore, *T-PTM is an inverse semi-group.*
- Using inverse semi-group theory, various properties of T-PTM are immediate (see paper).

Recursive and Infinite PTM

- To *render* a temporal value, one needs to incrementally enumerate its instantaneous values over time.
- With PTM, this is straightforward, even for recursively defined, infinite values:

  `m = c 4 en :+: m`
- Even parallel composition is OK:

  `m1 :=: m2`
  - Render m1
  - Render m2
  - Time-merge the results

Recursive and Infinite Tiles

- But there is a problem with:
  x = t c 4 en % x
  because *the value of post is infinite*.
- Even this version is problematical:
  x = t c 4 en % re x
- In general we cannot always render t1 % t2 because *the anacrusis of t2 may begin before* pre t1.
- By defining a new operator %\ that *ignores* such an anacrusis, we make progress in that:
  x = t c 4 en %\ re x
  can be rendered properly.
- But note: %\ *is not associative*.

In the paper…

- Further exploration of recursive tiles of form t = f t.
  - Relies on special fixpoint operator
- Implementation of T-PTM in Euterpea (PTM)
  - Serves as specification
- Observational equivalence.
- Formal connection between (:+:,:=:) and %.
- Other operators: resynch, coresynch, stretch, costretch.
- Examples.

Thank You!

Any questions?