

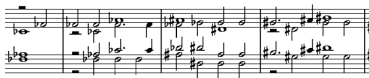
# Representation of musical notation in Haskell

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# Motivation



- (Notated) musical pitches are the points on a lattice
- (Notated) musical intervals connect the points, forming a ‘free Abelian group’ with two generators

# Theory

a#

b#

c×

d×

a♯

b♯

c#

d#

e#

f×

g×

a♭

b♭

c♯

d♯

e♯

f#

g#

A#

B#

b♭♭

c♭

d♭

e♭

f♯

g♯

A♯

B♯

d♭♭

e♭♭

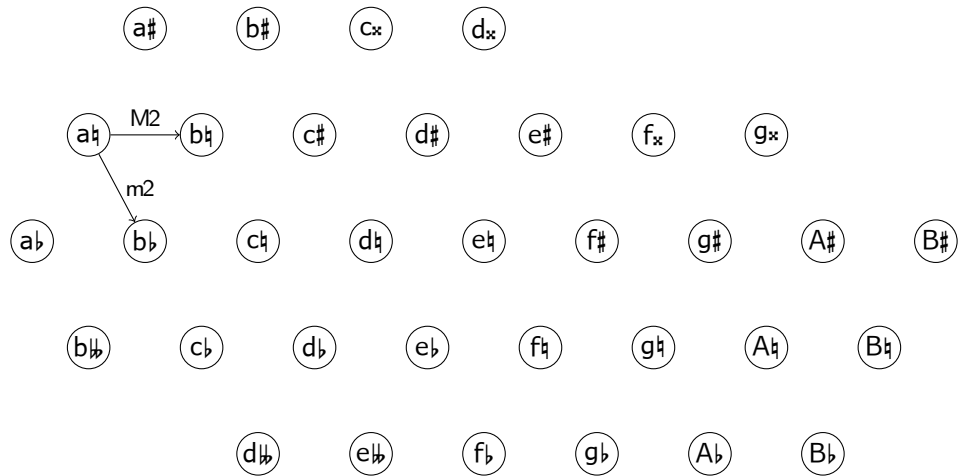
f♭

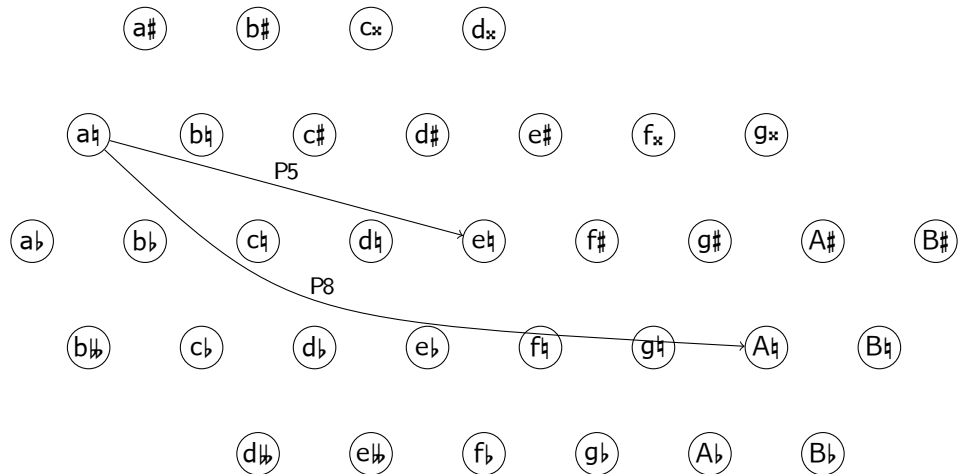
g♭

A♭

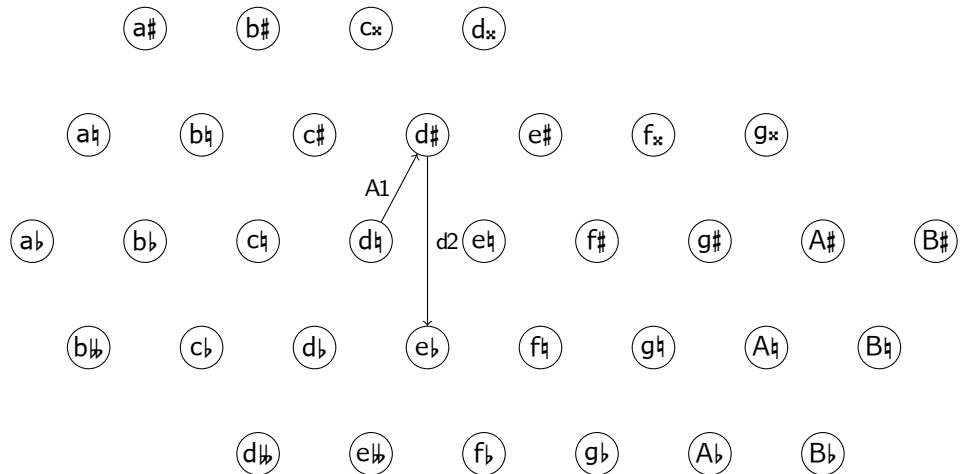
B♭

# Theory





# Theory



- ‘Syntonic’ temperaments assign two frequency ratios to the two generators
- ‘Equal’ temperaments project the two dimensions down to one



- Pythagorean

$$P5 \rightarrow 3/2, P8 \rightarrow 2$$

- 'Quarter-comma meantone'

$$M3 \rightarrow 5/4, P8 \rightarrow 2$$

- Pythagorean

$$P5 \rightarrow 3/2, P8 \rightarrow 2$$

- 'Quarter-comma meantone'

$$M3 \rightarrow 5/4, P8 \rightarrow 2$$

- 12-equal temperament

$$d2 \rightarrow 1, P8 \rightarrow 2$$

- 19-equal temperament

$$dd2 \rightarrow 1, P8 \rightarrow 2$$

- 31-equal temperament

$$dddd3 \rightarrow 1, P8 \rightarrow 2$$

# Implementation

- Flexibility (via typeclasses) in what counts as a `Pitch`, `Interval` or `Duration`
- A `Note` is an ordered pair (`Pitch`, `Duration`)
- A `Phrase` is just a linked list, `[Note]`
- A piece of music consists of a Rose tree of musical phrases
- Internally the preferred lattice basis is  $(A1, d2)$
- Have to invert a  $2 * 2$  matrix to calculate tuning map