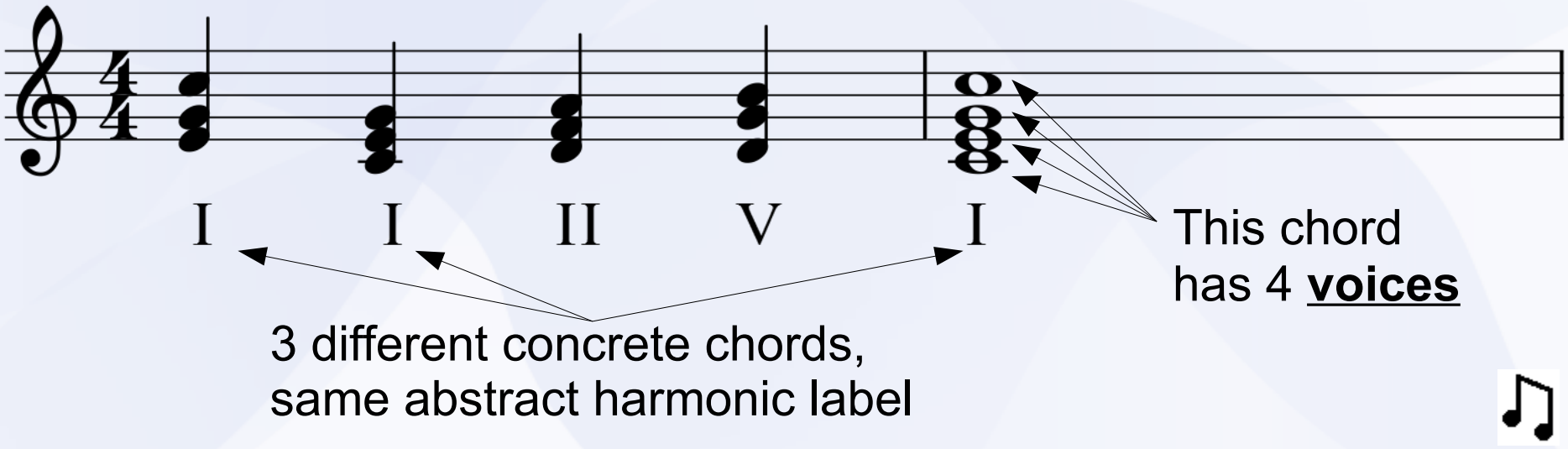
Grammar-Based Automated Music Composition in Haskell

Donya Quick and Paul Hudak
Yale University
Department of Computer Science

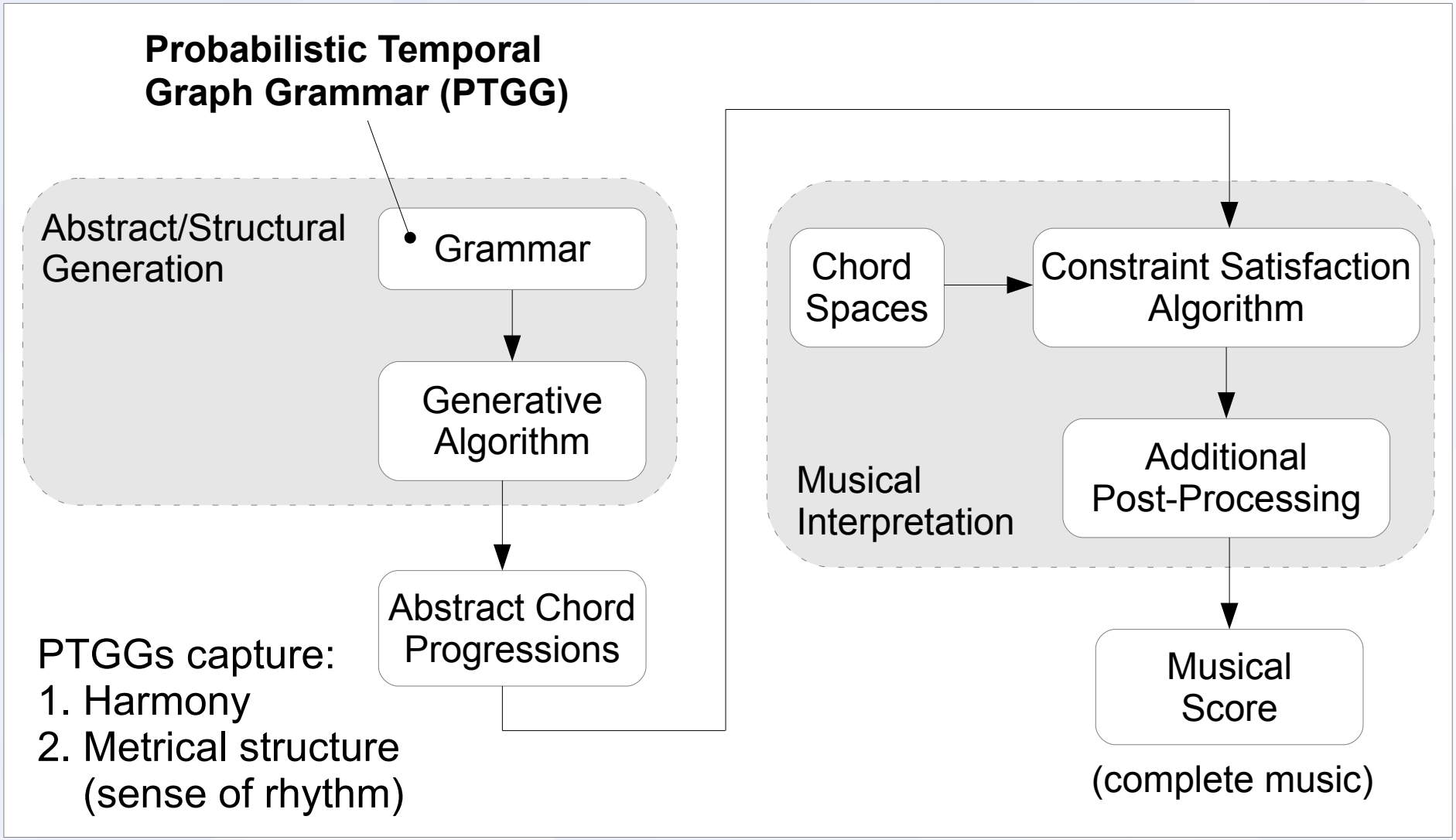
FARM 2013

All You Need To Know About Music

- A **chord** is a collection of simultaneous pitches.
  - Roman numerals `I` - `VII` are **abstract chords**.
    - Many ways to interpret them musically.
    - Interpretation depends on **key/mode**.
  - **Concrete chords** are what appear on scores.



Composition System Overview



Probabilistic Temporal Graph Grammar (PTGG): Alphabet and Notations

- Chords in the grammar are Roman numerals*:*
  $C = \{\mathtt{I}, \mathtt{II}, \mathtt{III}, \mathtt{IV}, \mathtt{V}, \mathtt{II}, \mathtt{VII}\}$
- c^t is the chord c with duration t (any real number).
- A chord progression is written: $c_1^{t1}\ c_2^{t2}\ c_3^{t3}\ \ldots\ c_n^{tn}$
- Modulations: $M = \{\mathtt{M}_2, \mathtt{M}_3, \mathtt{M}_4, \mathtt{M}_5, \mathtt{M}_6, \mathtt{M}_7\}$
  - Modulations change a section's key/mode.
  - Parentheses are used to denote modulated sections: $m(c_1^{t1} \ldots c_n^{tn})$, where $m \in M$.
    - *Parentheses are a "meta-symbol"*

PTGG Definition: $G = (N,T,R,S)$

- Nonterminals: $N = \{c^t \mid c \in C, t \text{ is a real number}\}$
- Terminals: $N \cup M$ These are infinite sets!
- Start symbol $S = \mathrm{I}^t$, where t is total duration desired.
- Rules are **functions** of duration from chords to chord progressions: $c^t \rightarrow f(t)$. For example:

(0.2) $\mathrm{I}^t \rightarrow \mathrm{V}^{t/2}\ \mathrm{I}^{t/2}$

(0.1) $\mathrm{V}^t \rightarrow \mathrm{M}_5(\mathrm{I}^t)$

(0.1) $\mathrm{V}^t \rightarrow \mathrm{V}^t$

(0.1) $\mathrm{I}^t \rightarrow$ **let** x=$\mathrm{I}^{t/2}$ **in** x x

Probability of application

Both instances of x must be identical after generation.

let x = $\mathrm{V}^{t1}\ \mathrm{I}^{t2}$ **in** x x $\Rightarrow$ $\mathrm{V}^{t1}\ \mathrm{I}^{t2}\ \mathrm{V}^{t1}\ \mathrm{I}^{t2}$

Recall: $C = \{\mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}, \mathrm{V}, \mathrm{II}, \mathrm{VII}\}$ and $M = \{\mathrm{M}_2, \mathrm{M}_3, \mathrm{M}_4, \mathrm{M}_5, \mathrm{M}_6, \mathrm{M}_7\}$

Haskell Implementation: Progressions

```
data CType = I | II | III | IV | V | VI | VII
    deriving (Eq, Show, Ord, Enum)
```

$C = \{\texttt{I}, \texttt{II}, \ldots, \texttt{VII}\}$

```
data MType = M2 | M3 | M4 | M5 | M6 | M7
    deriving (Eq, Show, Ord, Enum)
```

$M = \{M_1, \ldots, M_5\}$

```
data Chord = Chord Dur CType
    deriving (Eq, Show)
```

$\texttt{I}^t$ becomes *Chord t I*

```
data Term =
    NT Chord | S [Term] | Mod MType Term |
    Let String Term Term | Var String
     deriving (Eq, Show)
```

let *x* = *A* **in** *B*
becomes
Let "x" A B

$\texttt{V}^{t1}\ \texttt{I}^{t2}$ becomes *S [Chord t1 V, Chord t2 I]*

Haskell Implementation: Rules

```
type Prob = Double
type RuleFun = Dur → Term
data Rule = (CType, Prob) :→ RuleFun
```

Shorthand functions:

```
i  t = Chord t I   :: RuleFun
ii t = Chord t II :: RuleFun
```

etc.

```
r1 = (I, 0.2) :→ i
```

$\mathbb{I}^t \longrightarrow \mathbb{I}^t$

```
r2 = (I, 0.2) :→
     λt → S [v (t/2), i (t/2)]
```

$\mathbb{I}^t \longrightarrow \mathbb{V}^{t/2} \quad \mathbb{I}^{t/2}$

```
r3 = (V, 0.10) :→ ( Mod M5 . i )
```

$\mathbb{V}^t \longrightarrow M_5(\mathbb{I}^t)$

```
r4 = (I, 0.1) :→ λt →
     Let "x" (i (t/2))  S [Var "x", Var "x"]
```

$\mathbb{I}^t \longrightarrow$ **let** $x = \mathbb{I}^{t/2}$ **in** $x\ x$

Example of Generative Algorithm

Rules Applied (Stochastic)

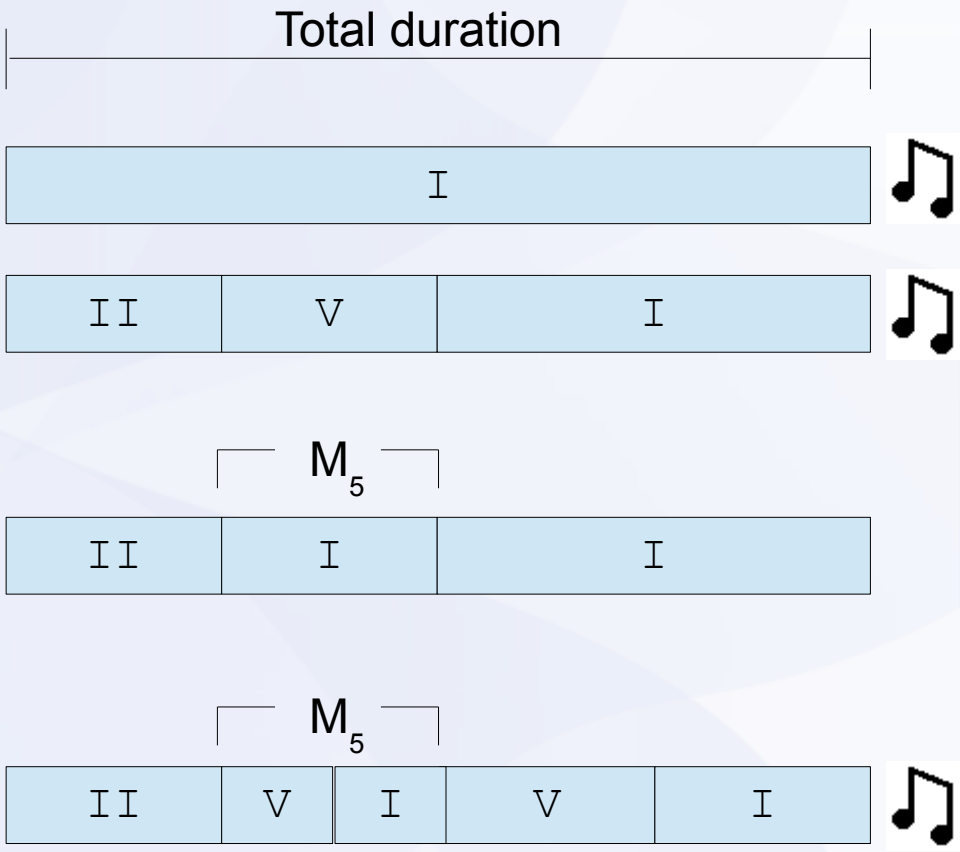
Start symbol: $\mathtt{I}^t$

$\mathtt{I}^t \rightarrow \mathtt{I}\,\mathtt{I}^{t/4}\ \mathtt{V}^{t/4}\ \mathtt{I}^{t/2}$

id, $\mathtt{V}^t \rightarrow \mathtt{M}_5\ (\mathtt{I}^t)$, id

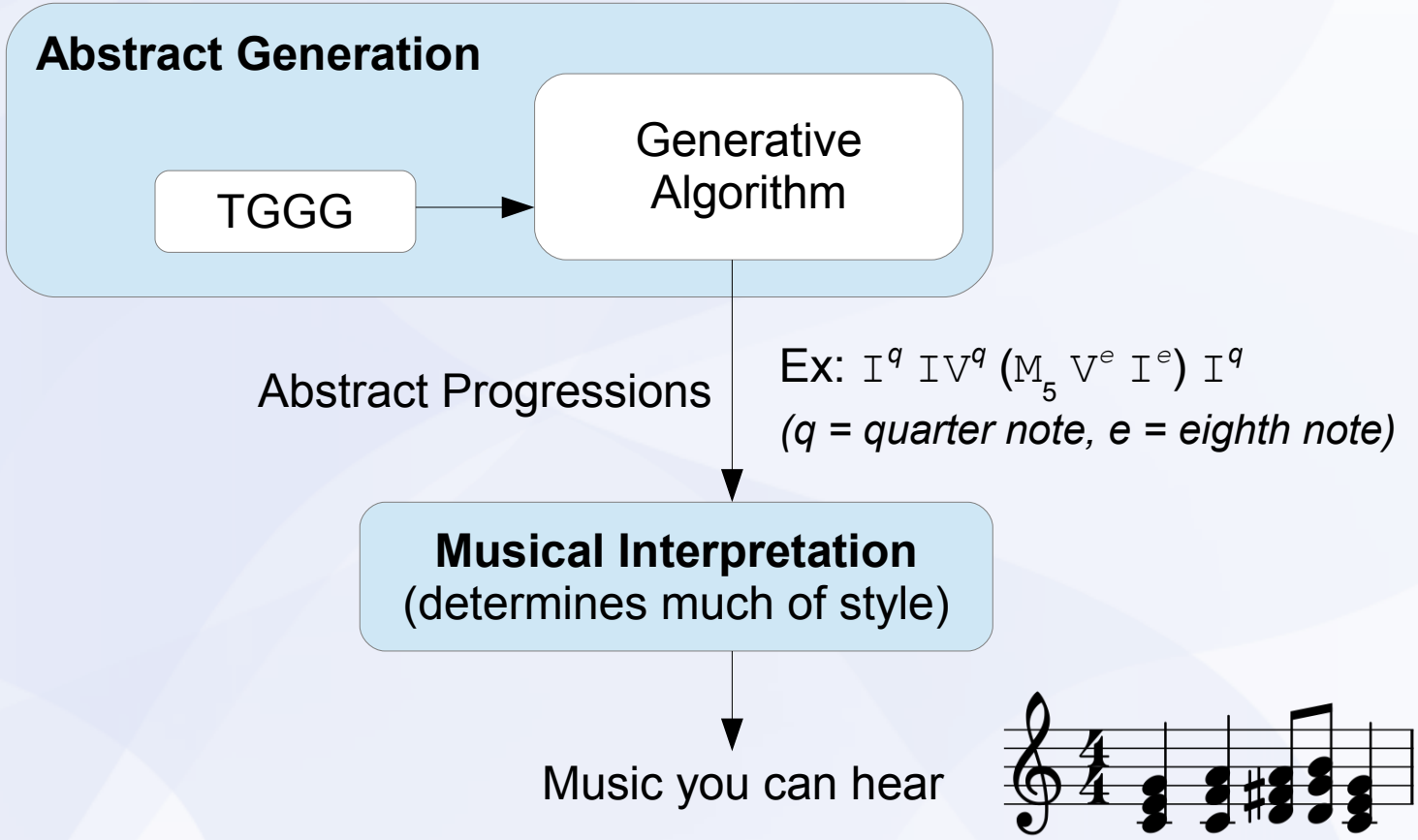
(id means $c^t \rightarrow c^t$)

id, $\mathtt{I}^t \rightarrow \mathtt{V}^{t/2}\ \mathtt{I}^{t/2}$, $\mathtt{I}^t \rightarrow \mathtt{V}^{t/2}\ \mathtt{I}^{t/2}$



For **let** $x = A$ **in** B, the phrases A and B are generated separately, leaving instances of $x \in B$ unaltered. Then, instances of x can be instantiated.

Musical Interpretation



We use **chord spaces** as an integral part of our interpretation.

Chord Spaces

- Mathematically grouping chords in musically useful ways.
  - Each chord belongs to an **equivalence class**.
- Examples generated with classical chord spaces [1,2] and also “jazz spaces.”
- Assigning pitches to Roman numerals reduces to a path-finding and **constraint-satisfatction** problem [3].
  - For each **abstract** chord, choose a **concrete** chord from its equivalence class meeting some criteria.
  - *Let* constraints shrink the search space!

[1] C. Callender et al., “Generalized voice-leading spaces,” Science Magazine 2008.
[2] D. Tymoczko et al., “The geometry of musical chords.” Science Magazine, 2006.
[3] D. Quick and P. Hudak, “Computing with chord spaces,” ICMC 2012.

Let Constraints and Chord Spaces

- Progression: **let** x = P Q **in** x x
  $\Rightarrow$ P Q P Q

P & Q are **abstract** chords, like `I` or `V`

- Chord space: P~{a,b}, Q~{c,d}
  Imposed ordering/indices: 0 1 0 1

a, b, c, & d are **concrete** chords

Depth first without lets:

#	Ind.	Value	Satisfies Lets?
1	0000	acac	Yes
2	0001	acad	No
3	0010	acbc	No
4	0011	acbd	No
5	0100	adac	No
...	…	…	…
64	1111	bdbd	Yes

Depth first WITH lets:

#	Ind.	Value	Satisfies Lets?
1	0000	acac	Yes
2	0101	adad	Yes
3	1010	bcbc	Yes
4	1111	bdbd	Yes

Constrained indices move in lock-step, dramatically reducing the number of solutions explored.

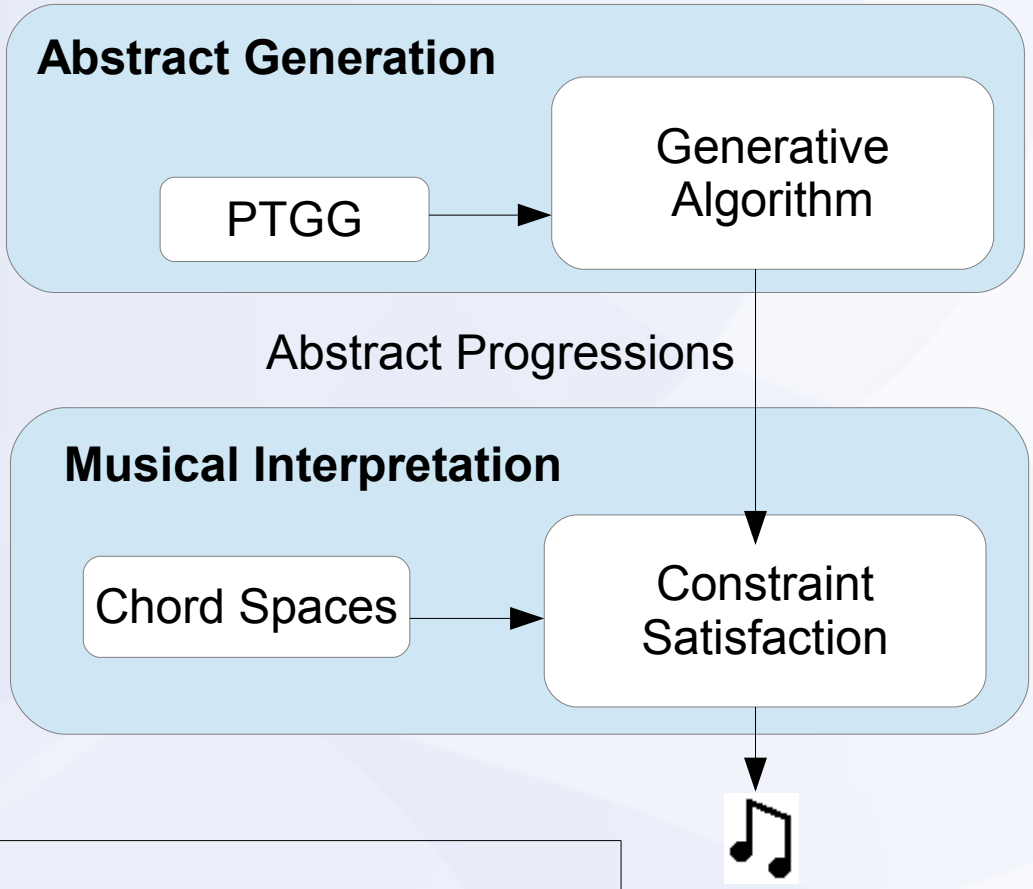
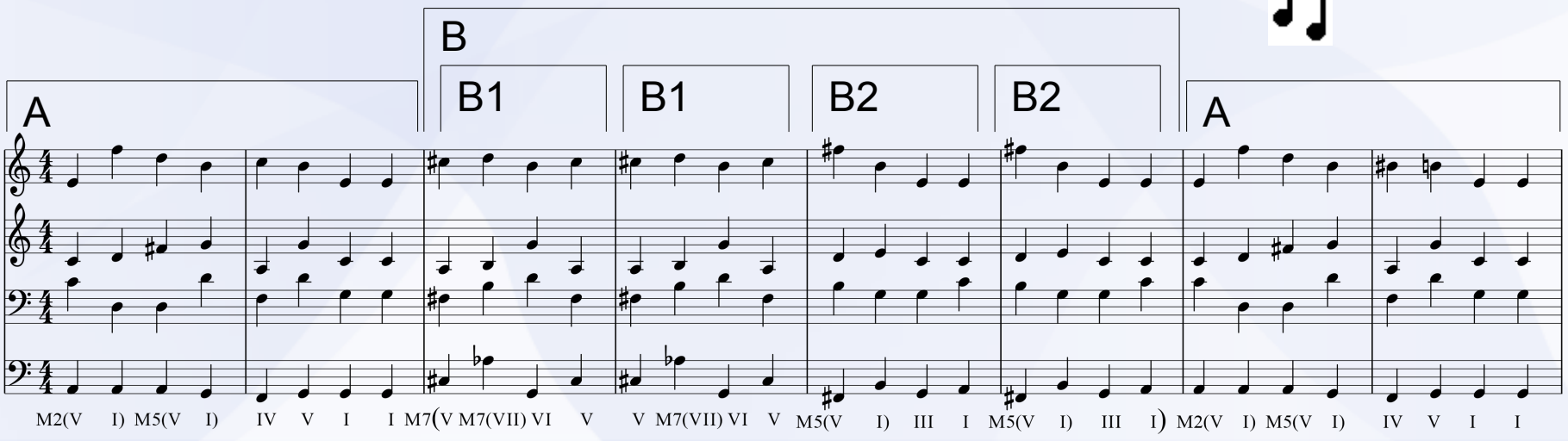
System #1

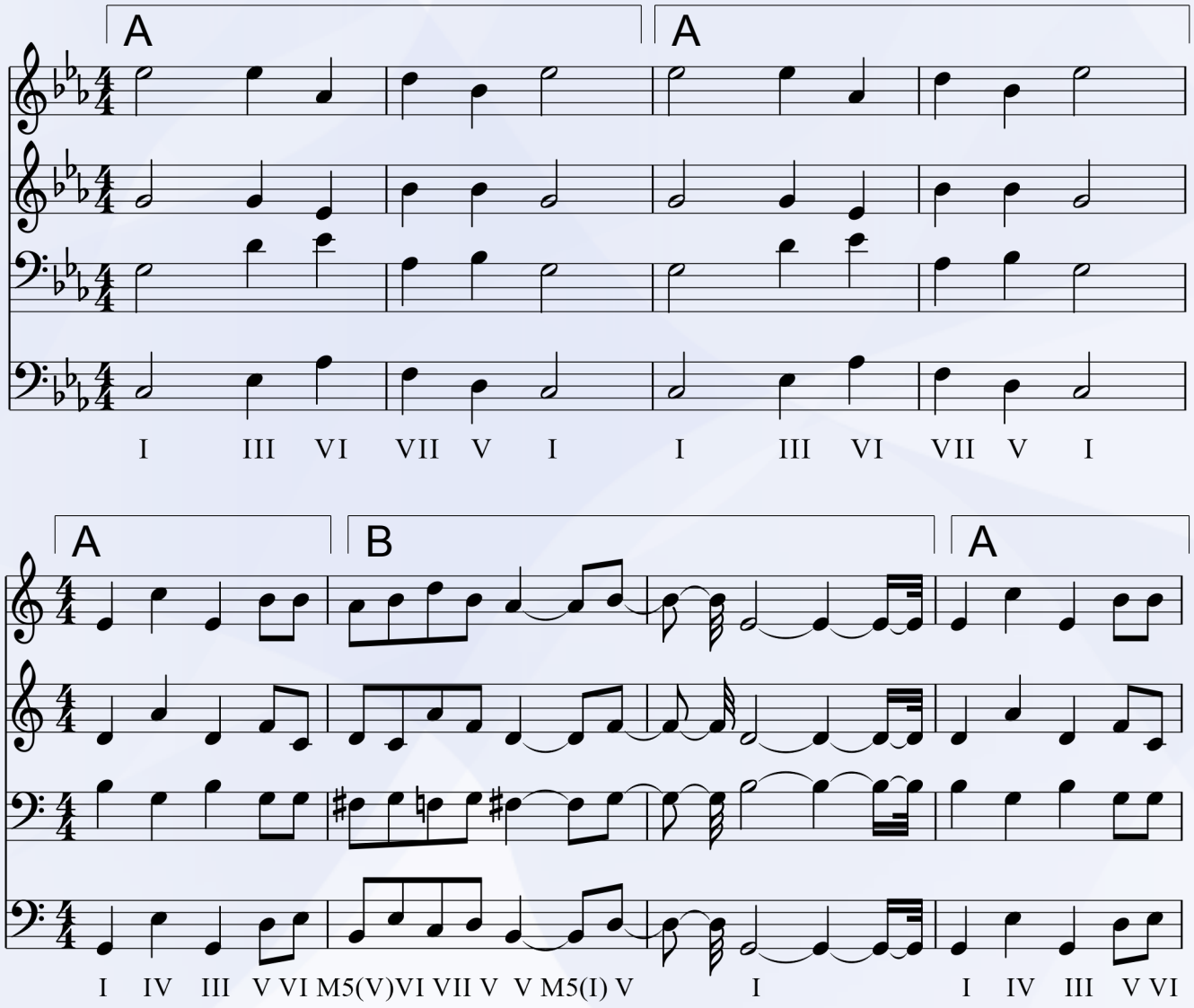
8-measure example.

Classical chord space for 4 voices.

Shows repetition from nested Let expressions.

NO extra musical post-processing!



Same System, More Examples

Classical chord spaces

Jazz chord spaces with a syncopated rule set.

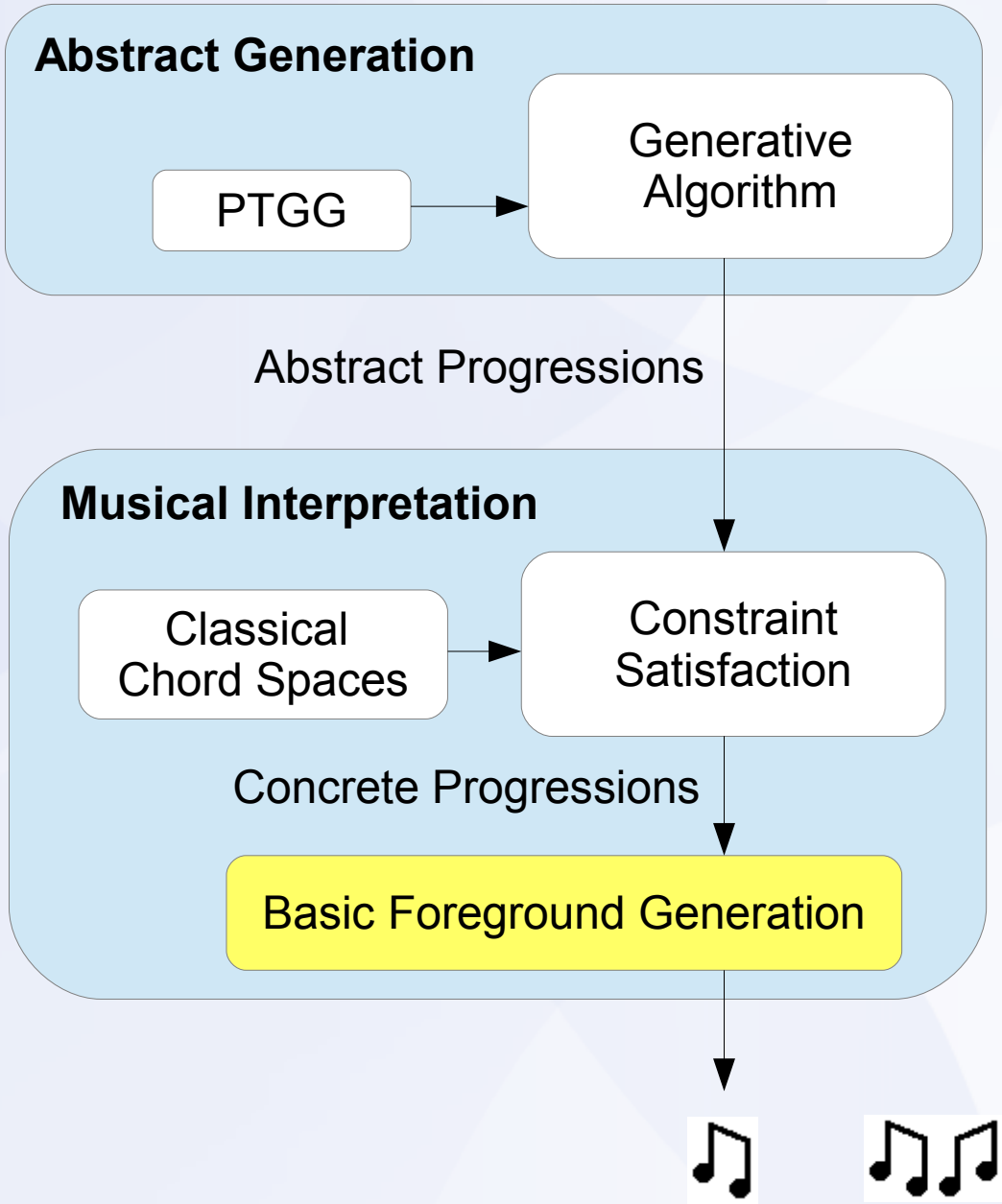
System #2

Simple Classical Music

Uses classical chord spaces for 4 voices.

Foreground features added include passing and neighboring tones.

Bach chorale for comparison:



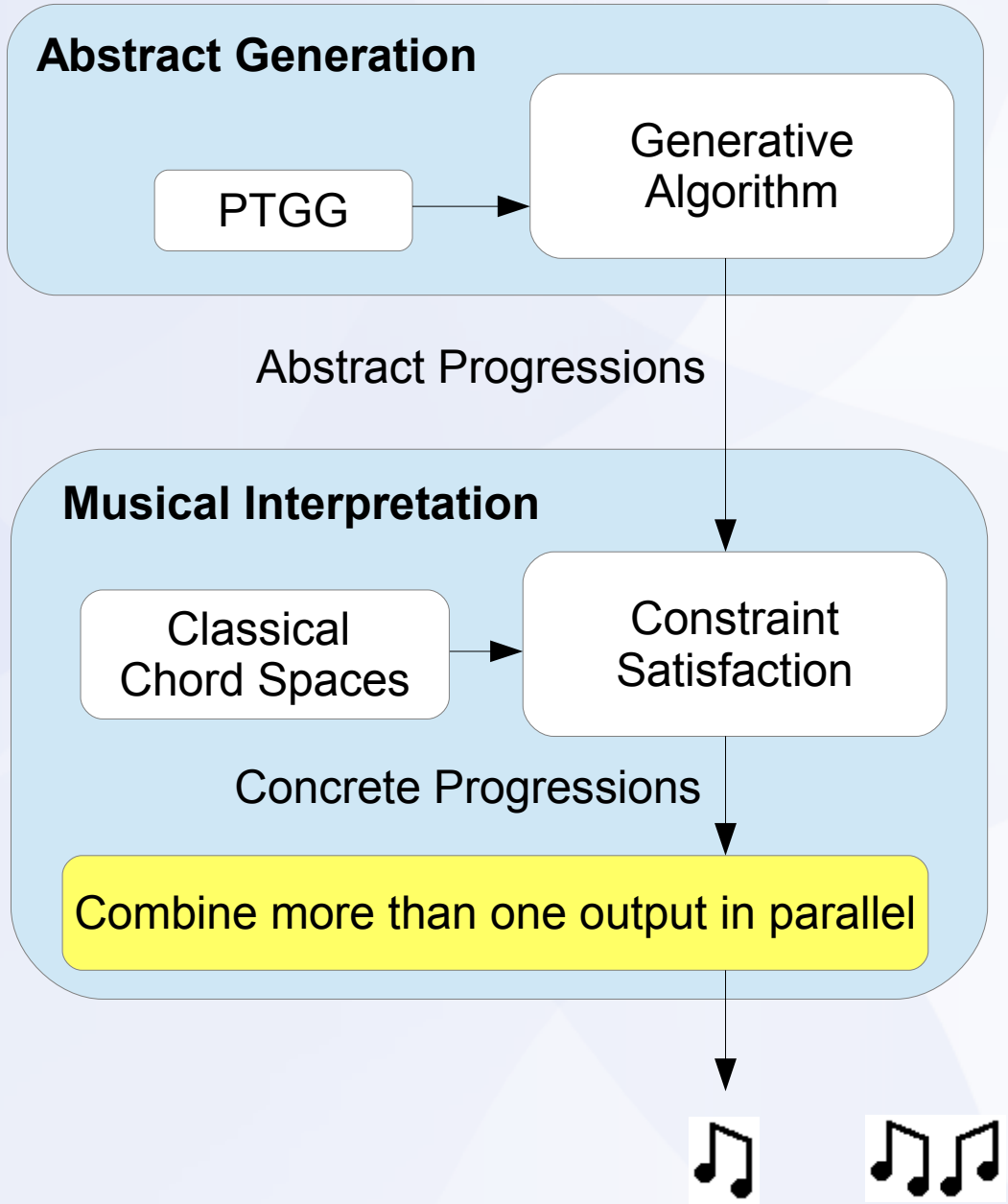
System #3

Modern, texturally interesting music

Uses classical chord spaces for 4 voices.

Parts were generated independently and later combined.

Human-controlled: volume changes, staggering of voices, choice of seeds

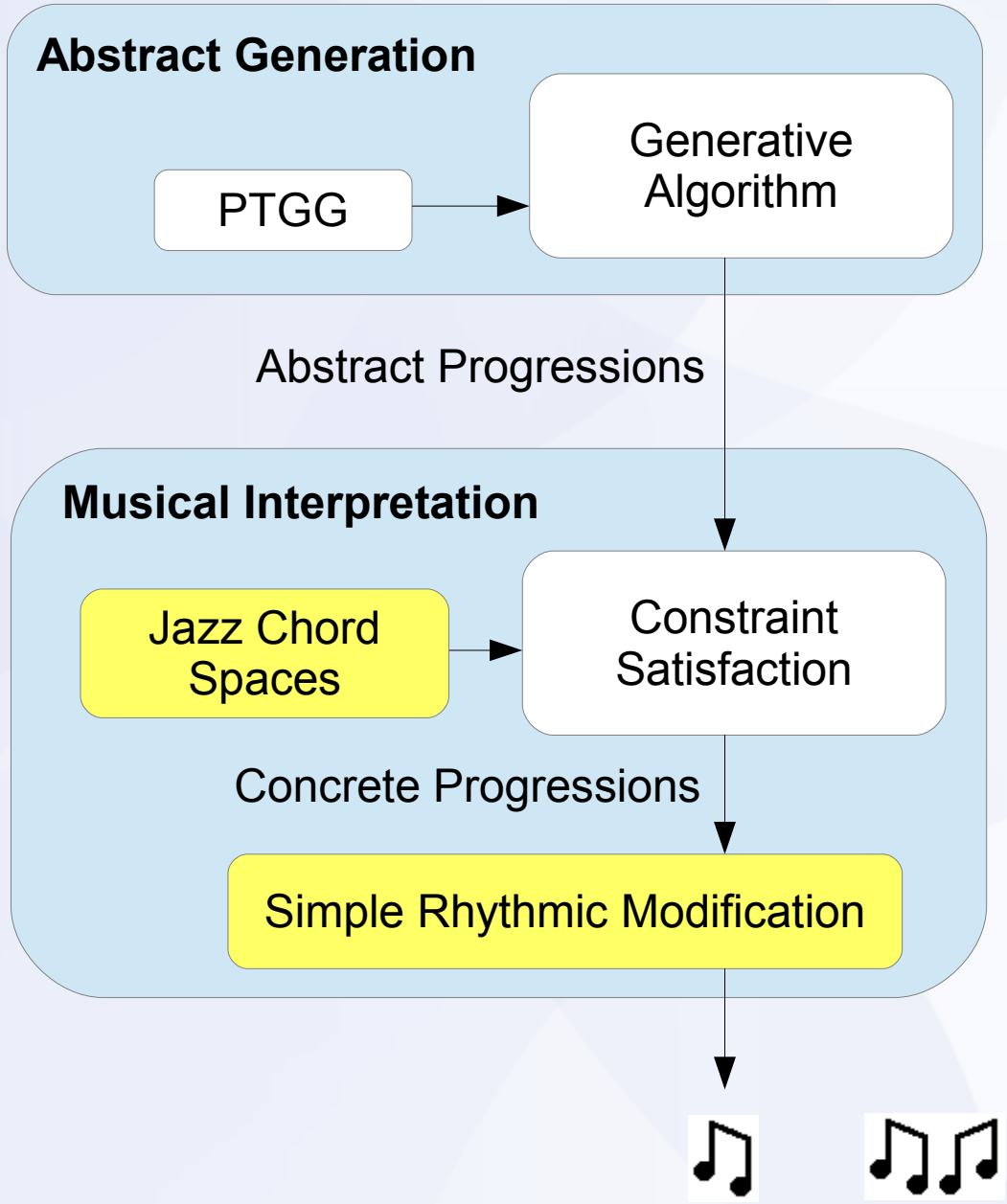


System #4

Jazz Harmonies

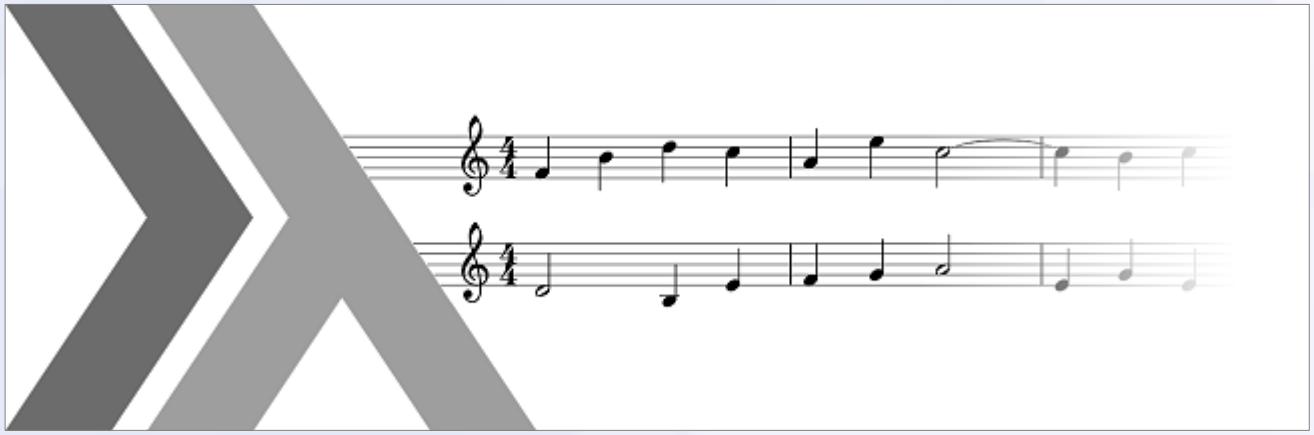
Jazz chord spaces add seconds and sevenths for 5 voices.

Lowest voice's rhythm was stochastically altered.



Conclusions

- A functional approach to modeling music gives us:
  - An elegant Haskell implementation.
  - *Let* expressions that support repetition of phrases.
- Chord spaces allow many different musical styles.
- Areas of potential future work:
  - Melody – currently left to post-processing.
  - More diverse rhythmic support (3/4, triplets in 4/4, etc.)
  - Larger-scale/more complex developmental patterns
    - Theme and variations, partial repetition, etc.
  - Empirical testing with human subjects.
    - How well is a particular style reproduced?

Thank You!

- Implementation at: `haskell.cs.yale.edu`
- Full recordings of examples at: `soundcloud.com/donyaquick`

  Monadic Algorithm Compositions 1, 2, and 3

Complete Rule Set

Num.	Probability	Rule
1	0.20	$I^t \rightarrow II^{t/4}\ V^{t/4}\ I^{t/2}$
2	0.20	$I^t \rightarrow I^{t/4}\ IV^{t/4}\ V^{t/4}\ I^{t/4}$
3	0.20	$I^t \rightarrow V^{t/2}\ I^{t/2}$
4	0.20	$I^t \rightarrow I^{t/4}\ II^{t/4}\ V^{t/4}\ I^{t/4}$
5	0.20	$I^t \rightarrow I^t$
6	0.80	$II^t \rightarrow II^t$
7	0.20	$II^t \rightarrow (M_2\ V^{t/2}\ I^{t/2})$
8	0.70	$III^t \rightarrow III^t$
9	0.30	$III^t \rightarrow (M_3\ I^t)$
10	0.80	$IV^t \rightarrow IV^t$
11	0.20	$IV^t \rightarrow (M_4\ I^{t/4}\ V^{t/4}\ I^{t/2})$
12	0.10	$V^t \rightarrow V^t$
13	0.15	$V^t \rightarrow IV^{t/2}\ V^{t/2}$
14	0.10	$V^t \rightarrow III^{t/2}\ VI^{t/2}$
15	0.10	$V^t \rightarrow I^{t/4}\ III^{t/4}\ VI^{t/4}\ I^{t/4}$

Num.	Probability	Rule
16	0.10	$V^t \rightarrow V^{t/4}\ VI^{t/4}\ VII^{t/4}\ V^{t/4}$
17	0.10	$V^t \rightarrow V^{t/2}\ VI^{t/2}$
18	0.10	$V^t \rightarrow III^t$
19	0.05	$V^t \rightarrow (M_7\ V^t)$
20	0.10	$V^t \rightarrow VII^t$
22	0.10	$V^t \rightarrow (M_5\ I^t)$
22	0.70	$VI^t \rightarrow VI^t$
23	0.30	$VI^t \rightarrow (M_6\ I^t)$
24	0.40	$VII^t \rightarrow VII^t$
25	0.50	$VII^t \rightarrow I^{t/2}\ III^{t/2}$
26	0.10	$VII^t \rightarrow (M_7\ I^t)$

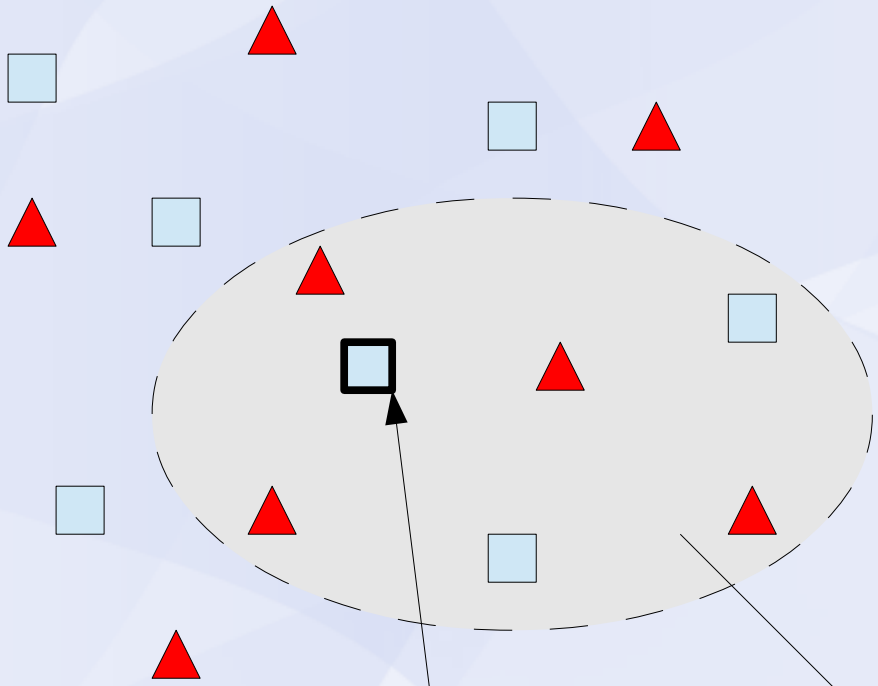
Extra Let rules for all $c \in C$:

$c^t \rightarrow$ let x = $c^{t/2}$ in x x

$c^t \rightarrow$ let x = $c^{t/4}$ in x $c^{t/2}$ x

$c^t \rightarrow$ let x = $c^{t/4}$ in x $\nabla^{t/2}$ x

Voice-Leading with Chord Spaces



Input chords (abstract):
Cmaj, Gmaj, …

Cmaj chords

Gmaj chords

Constraints (concrete)

Next chords to choose from

Suppose we pick this Cmaj chord