Grammar-Based Automated Music Composition in Haskell

Donya Quick and Paul Hudak
Yale University
Department of Computer Science

FARM 2013
All You Need To Know About Music

- A **chord** is a collection of simultaneous pitches.
  - Roman numerals I - VII are **abstract chords**.
    - Many ways to interpret them musically.
    - Interpretation depends on **key/mode**.
  - **Concrete chords** are what appear on scores.

This chord has 4 **voices**

3 different concrete chords, same abstract harmonic label
Composition System Overview

Probabilistic Temporal Graph Grammar (PTGG)

Abstract/Structural Generation
- Grammar
  - Generative Algorithm
    - Abstract Chord Progressions

PTGGs capture:
1. Harmony
2. Metrical structure (sense of rhythm)

Chord Spaces
Constraint Satisfaction Algorithm
Musical Interpretation
Additional Post-Processing
Musical Score
(complete music)
Probabilistic Temporal Graph Grammar (PTGG): Alphabet and Notations

- Chords in the grammar are Roman numerals: $C = \{I, II, III, IV, V, II, VII\}$
- $c^t$ is the chord $c$ with duration $t$ (any real number).
- A chord progression is written: $c_1^{t_1} c_2^{t_2} c_3^{t_3} \ldots c_n^{t_n}$
- Modulations: $M = \{M_2, M_3, M_4, M_5, M_6, M_7\}$
  - Modulations change a section's key/mode.
  - Parentheses are used to denote modulated sections: $m(c_1^{t_1} \ldots c_n^{t_n})$, where $m \in M$.

- Parentheses are a “meta-symbol”
PTGG Definition: $G = (N,T,R,S)$

- Nonterminals: $N = \{c^t \mid c \in C, t$ is a real number\}
- Terminals: $N \cup M$ These are infinite sets!
- Start symbol $S = I^t$, where $t$ is total duration desired.
- Rules are **functions** of duration from chords to chord progressions: $c^t \rightarrow f(t)$. For example:
  
  \[
  \begin{align*}
  (0.2) & \ I^t \rightarrow V^{t/2} \ I^{t/2} \\
  (0.1) & \ V^t \rightarrow M_5(I^t) \\
  (0.1) & \ V^t \rightarrow V^t \\
  (0.1) & \ I^t \rightarrow \text{let } x=I^{t/2} \text{ in } x \ x 
  \end{align*}
  \]

Recall: $C = \{I, II, III, IV, V, VI, VII\}$ and $M = \{M_2, M_3, M_4, M_5, M_6, M_7\}$
Haskell Implementation: Progressions

data CType = I | II | III | IV | V | VI | VII
deriving (Eq, Show, Ord, Enum)

data MType = M2 | M3 | M4 | M5 | M6 | M7
deriving (Eq, Show, Ord, Enum)

data Chord = Chord Dur CType
deriving (Eq, Show)

data Term =
  NT Chord | S [Term] | Mod MType Term | Let String Term Term | Var String
deriving (Eq, Show)

C = \{I, II, ..., VII\}

M = \{M_1, ..., M_5\}

I_t \text{ becomes } Chord t I

let x = A in B becomes Let “x” A B

\forall t_1 I_t^2 \text{ becomes } S [Chord t_1 V, Chord t_2 I]
Haskell Implementation: Rules

type 
Prob = Double

type 
RuleFun = Dur → Term

data 
Rule = (CType, Prob) :→ RuleFun

r1 = (I, 0.2) :→ i

r2 = (I, 0.2) :→ λt → S [ν (t/2), i (t/2)]

r3 = (V, 0.10) :→ (Mod M5 . i )

r4 = (I, 0.1) :→ λt → Let “x” (i (t/2)) S [Var “x”, Var “x”]

Shorthand functions:
i t = Chord t I :: RuleFun
ii t = Chord t II :: RuleFun
etc.

\[\begin{align*}
\mathbb{I}^t & \rightarrow \mathbb{I}^t \\
\mathbb{I}^t & \rightarrow \mathbb{V}^{t/2} \mathbb{I}^{t/2} \\
\mathbb{V}^t & \rightarrow M_5(\mathbb{I}^t) \\
\mathbb{I}^t & \rightarrow \text{let } x=\mathbb{I}^{t/2} \text{ in } x \times x
\end{align*}\]
### Example of Generative Algorithm

**Rules Applied (Stochastic)**

**Start symbol:** $I^t$

- $I^t \rightarrow II^{t/4} V^{t/4} I^{t/2}$

- $id, V^t \rightarrow M_5 (I^t), id$

  (id means $c^t \rightarrow c^t$)

- $id, I^t \rightarrow V^{t/2} I^{t/2}, I^t \rightarrow V^{t/2} I^{t/2}$

---

**Total duration**

<table>
<thead>
<tr>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$II$</td>
</tr>
<tr>
<td>$M_5$</td>
</tr>
<tr>
<td>$II$</td>
</tr>
<tr>
<td>$M_5$</td>
</tr>
<tr>
<td>$II$</td>
</tr>
</tbody>
</table>

For **let** $x = A$ **in** $B$, the phrases $A$ and $B$ are generated separately, leaving instances of $x \in B$ unaltered. Then, instances of $x$ can be instantiated.
We use **chord spaces** as an integral part of our interpretation.
Chord Spaces

- Mathematically grouping chords in musically useful ways.
  - Each chord belongs to an equivalence class.
- Examples generated with classical chord spaces [1,2] and also “jazz spaces.”
- Assigning pitches to Roman numerals reduces to a path-finding and constraint-satisfaction problem [3].
  - For each abstract chord, choose a concrete chord from its equivalence class meeting some criteria.
  - Let constraints shrink the search space!

Let Constraints and Chord Spaces

- **Progression:** \( \text{let } x = P \ Q \ \text{in } x \ x \Rightarrow P \ Q \ P \ Q \)

- **Chord space:** \( P \sim \{a,b\}, \ Q \sim \{c,d\} \)

Imposed ordering/indices: \( 0 \ 1 \ 0 \ 1 \)

<table>
<thead>
<tr>
<th>Depth first without lets:</th>
<th>Depth first WITH lets:</th>
</tr>
</thead>
<tbody>
<tr>
<td>#</td>
<td>Ind.</td>
</tr>
<tr>
<td>---</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>0000</td>
</tr>
<tr>
<td>2</td>
<td>0001</td>
</tr>
<tr>
<td>3</td>
<td>0010</td>
</tr>
<tr>
<td>4</td>
<td>0011</td>
</tr>
<tr>
<td>5</td>
<td>0100</td>
</tr>
<tr>
<td>64</td>
<td>1111</td>
</tr>
</tbody>
</table>

Constrained indices move in lock-step, dramatically reducing the number of solutions explored.

\( P \) & \( Q \) are abstract chords, like \( I \) or \( V \)

\( a, \ b, \ c, \ \& \ d \) are concrete chords
System #1

8-measure example.

Classical chord space for 4 voices.

Shows repetition from nested Let expressions.

**NO extra musical post-processing!**
Same System, More Examples

Classical chord spaces

Jazz chord spaces with a syncopated rule set.
System #2
Simple Classical Music

Uses classical chord spaces for 4 voices.

Foreground features added include passing and neighboring tones.

Bach chorale for comparison:
System #3

Modern, texturally interesting music

Uses classical chord spaces for 4 voices.

Parts were generated independently and later combined.

Human-controlled: volume changes, staggering of voices, choice of seeds
System #4
Jazz Harmonies

Jazz chord spaces add seconds and sevenths for 5 voices.

Lowest voice's rhythm was stochastically altered.
Conclusions

• A functional approach to modeling music gives us:
  – An elegant Haskell implementation.
  – Let expressions that support repetition of phrases.
• Chord spaces allow many different musical styles.
• Areas of potential future work:
  – Melody – currently left to post-processing.
  – More diverse rhythmic support (3/4, triplets in 4/4, etc.)
  – Larger-scale/more complex developmental patterns
    • Theme and variations, partial repetition, etc.
  – Empirical testing with human subjects.
    • How well is a particular style reproduced?
Thank You!

- Implementation at: haskell.cs.yale.edu
- Full recordings of examples at: soundcloud.com/donyaquick

Monadic Algorithm Compositions 1, 2, and 3
## Complete Rule Set

<table>
<thead>
<tr>
<th>Num.</th>
<th>Probability</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.20</td>
<td>$I^t \rightarrow II^{t/4}$ $V^{t/4}$ $I^{t/2}$</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>$I^t \rightarrow I^{t/4}$ $IV^{t/4}$ $V^{t/4}$ $I^{t/4}$</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>$I^t \rightarrow V^{t/2}$ $I^{t/2}$</td>
</tr>
<tr>
<td>4</td>
<td>0.20</td>
<td>$I^t \rightarrow I^{t/4}$ $II^{t/4}$ $V^{t/4}$ $I^{t/4}$</td>
</tr>
<tr>
<td>5</td>
<td>0.20</td>
<td>$I^t \rightarrow I^t$</td>
</tr>
<tr>
<td>6</td>
<td>0.80</td>
<td>$II^t \rightarrow II^t$</td>
</tr>
<tr>
<td>7</td>
<td>0.20</td>
<td>$II^t \rightarrow (M_2 V^{t/2}$ $I^{t/2}$)</td>
</tr>
<tr>
<td>8</td>
<td>0.70</td>
<td>$III^t \rightarrow III^t$</td>
</tr>
<tr>
<td>9</td>
<td>0.30</td>
<td>$III^t \rightarrow (M_3 I^t)$</td>
</tr>
<tr>
<td>10</td>
<td>0.80</td>
<td>$IV^t \rightarrow IV^t$</td>
</tr>
<tr>
<td>11</td>
<td>0.20</td>
<td>$IV^t \rightarrow (M_4 I^{t/4}$ $V^{t/4}$ $I^{t/2})$</td>
</tr>
<tr>
<td>12</td>
<td>0.10</td>
<td>$V^t \rightarrow V^t$</td>
</tr>
<tr>
<td>13</td>
<td>0.15</td>
<td>$V^t \rightarrow IV^{t/2}$ $V^{t/2}$</td>
</tr>
<tr>
<td>14</td>
<td>0.10</td>
<td>$V^t \rightarrow III^{t/2}$ $VI^{t/2}$</td>
</tr>
<tr>
<td>15</td>
<td>0.10</td>
<td>$V^t \rightarrow I^{t/4}$ $III^{t/4}$ $VI^{t/4}$ $I^{t/4}$</td>
</tr>
<tr>
<td>16</td>
<td>0.10</td>
<td>$V^t \rightarrow V^{t/4}$ $VI^{t/4}$ $VII^{t/4}$ $V^{t/4}$</td>
</tr>
<tr>
<td>17</td>
<td>0.10</td>
<td>$V^t \rightarrow V^{t/2}$ $VI^{t/2}$</td>
</tr>
<tr>
<td>18</td>
<td>0.10</td>
<td>$V^t \rightarrow III^t$</td>
</tr>
<tr>
<td>19</td>
<td>0.05</td>
<td>$V^t \rightarrow (M_7 V^t)$</td>
</tr>
<tr>
<td>20</td>
<td>0.10</td>
<td>$V^t \rightarrow VII^t$</td>
</tr>
<tr>
<td>21</td>
<td>0.10</td>
<td>$V^t \rightarrow (M_5 I^t)$</td>
</tr>
<tr>
<td>22</td>
<td>0.70</td>
<td>$VI^t \rightarrow VI^t$</td>
</tr>
<tr>
<td>23</td>
<td>0.30</td>
<td>$VI^t \rightarrow (M_6 I^t)$</td>
</tr>
<tr>
<td>24</td>
<td>0.40</td>
<td>$VII^t \rightarrow VII^t$</td>
</tr>
<tr>
<td>25</td>
<td>0.50</td>
<td>$VII^t \rightarrow I^{t/2}$ $III^{t/2}$</td>
</tr>
<tr>
<td>26</td>
<td>0.10</td>
<td>$VII^t \rightarrow (M_7 I^t)$</td>
</tr>
</tbody>
</table>

---

Extra Let rules for all $c \in C$:

- $c^t \rightarrow $ let $x = c^{t/2}$ in $x$ $x$
- $c^t \rightarrow $ let $x = c^{t/4}$ in $x$ $c^{t/2}$ $x$
- $c^t \rightarrow $ let $x = c^{t/4}$ in $x$ $\forall^{t/2}$ $x$
Voice-Leading with Chord Spaces

Suppose we pick this Cmaj chord

Next chords to choose from

Input chords (abstract): Cmaj, Gmaj, ...

Cmaj chords

Gmaj chords

Constraints (concrete)