Grammar-Based Automated Music Composition in Haskell

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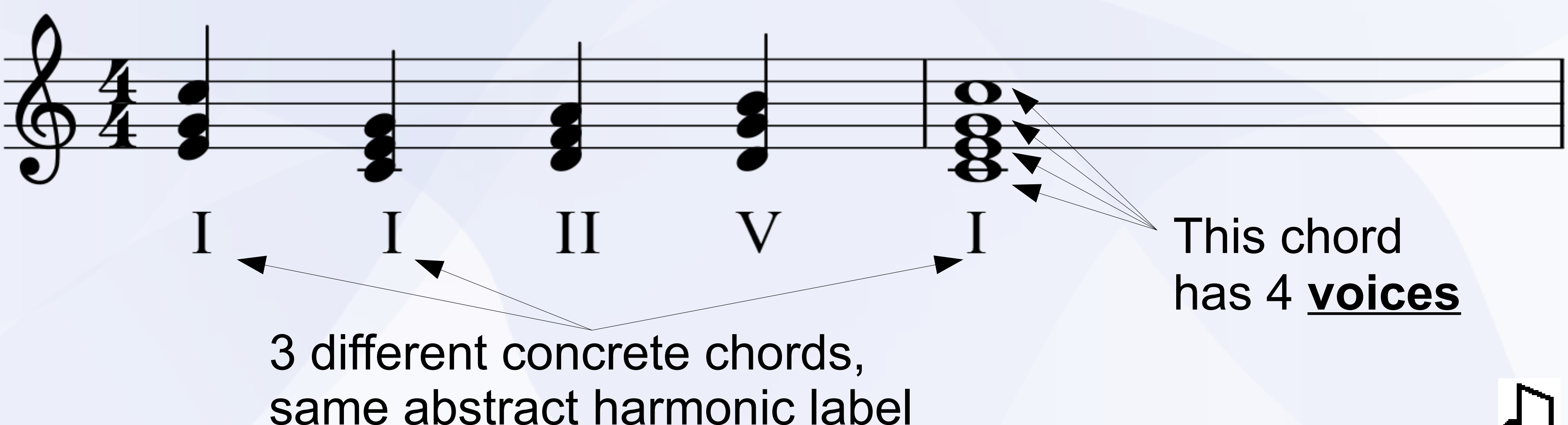
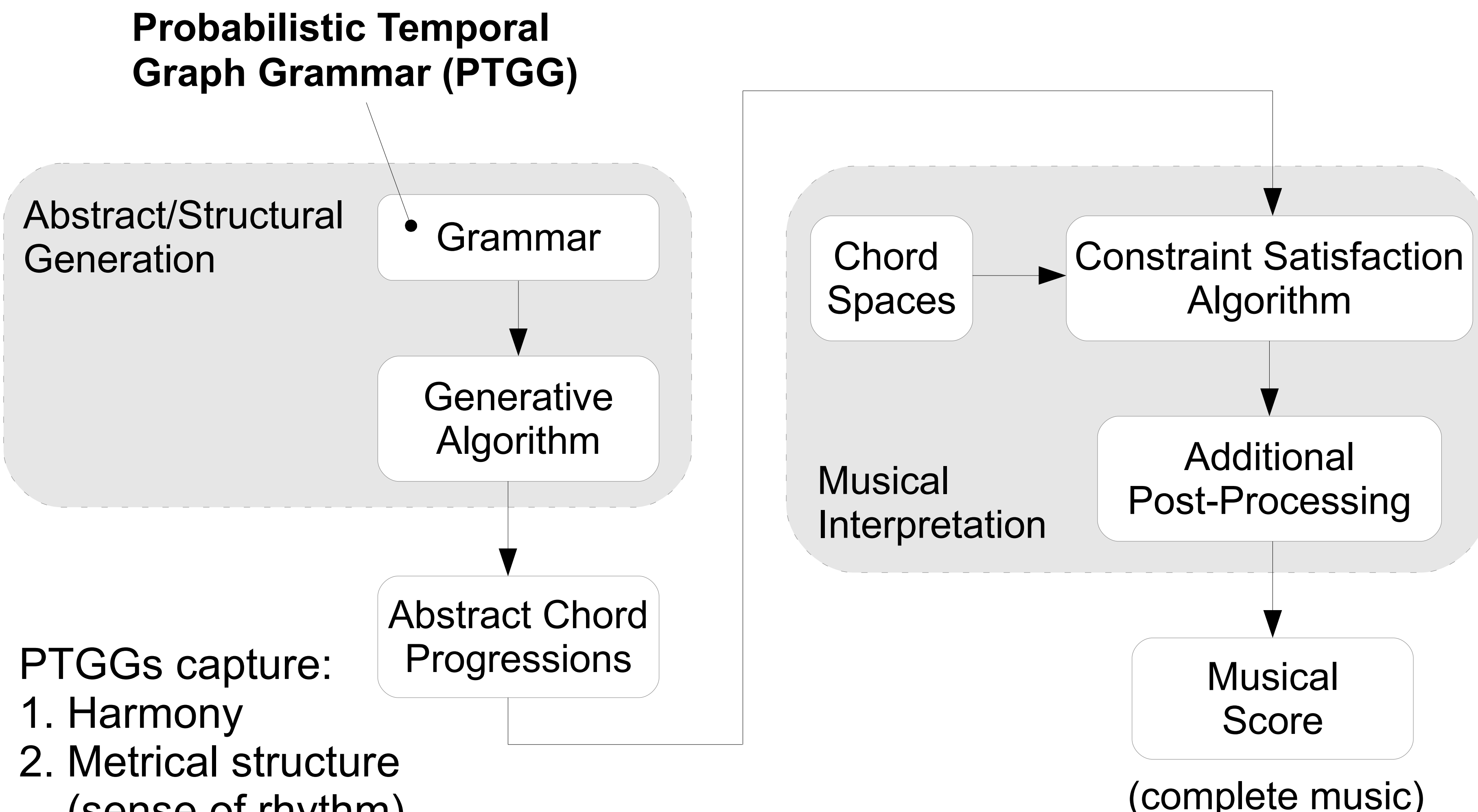
FARM 2013

All You Need To Know About Music

- A **chord** is a collection of simultaneous pitches.
  - Roman numerals `I` - `VII` are **abstract chords**.
    - Many ways to interpret them musically.
    - Interpretation depends on **key/mode**.
  - **Concrete chords** are what appear on scores.



Composition System Overview



PTGGs capture:
1. Harmony
2. Metrical structure
   (sense of rhythm)

Probabilistic Temporal Graph Grammar (PTGG): Alphabet and Notations

- Chords in the grammar are Roman numerals*:*
  C = {I, II, III, IV, V, II, VII}
- c^t is the chord c with duration t (any real number).
- A chord progression is written: $c_1^{t1}\ c_2^{t2}\ c_3^{t3}\ \ldots\ c_n^{tn}$
- Modulations: $M = \{M_2, M_3, M_4, M_5, M_6, M_7\}$
  - Modulations change a section's key/mode.
  - Parentheses are used to denote modulated sections: $m(c_1^{t1} \ldots c_n^{tn})$, where $m \in M$.
    - *Parentheses are a "meta-symbol"*

PTGG Definition: $G = (N,T,R,S)$

- Nonterminals: $N = \{c^t \mid c \in C, t \text{ is a real number}\}$
- Terminals: $N \cup M$

These are infinite sets!

- Start symbol $S = \mathrm{I}^t$, where t is total duration desired.
- Rules are **functions** of duration from chords to chord progressions: $c^t \rightarrow f(t)$. For example:

$$(0.2)\ \mathrm{I}^t \rightarrow \mathrm{V}^{t/2}\ \mathrm{I}^{t/2}$$
$$(0.1)\ \mathrm{V}^t \rightarrow \mathrm{M}_5(\mathrm{I}^t)$$
$$(0.1)\ \mathrm{V}^t \rightarrow \mathrm{V}^t$$
$$(0.1)\ \mathrm{I}^t \rightarrow \textbf{let } x=\mathrm{I}^{t/2} \textbf{ in } x\ x$$

Probability of application

Both instances of x must be identical after generation.

$\textbf{let } x = \mathrm{V}^{t1}\ \mathrm{I}^{t2} \textbf{ in } x\ x \Rightarrow \mathrm{V}^{t1}\ \mathrm{I}^{t2}\ \mathrm{V}^{t1}\ \mathrm{I}^{t2}$

Recall: $C = \{\mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}, \mathrm{V}, \mathrm{II}, \mathrm{VII}\}$ and $M = \{\mathrm{M}_2, \mathrm{M}_3, \mathrm{M}_4, \mathrm{M}_5, \mathrm{M}_6, \mathrm{M}_7\}$

Haskell Implementation: Progressions

data *CType* = *I* | *II* | *III* | *IV* | *V* | *VI* | *VII*
  deriving (*Eq*, *Show*, *Ord*, *Enum*)

C = {I, II, …, VII}

data *MType* = *M2* | *M3* | *M4* | *M5* | *M6* | *M7*
  deriving (*Eq*, *Show*, *Ord*, *Enum*)

M = $\{M_1, \ldots, M_5\}$

data *Chord* = *Chord Dur CType*
  deriving (*Eq*, *Show*)

I^t becomes *Chord t I*

data *Term* =
  NT Chord | *S* [*Term*] | *Mod MType Term* |
  Let String Term Term | *Var String*
  deriving (*Eq*, *Show*)

let *x* = *A* **in** *B*
becomes
Let "*x*" *A B*

V^{t1} I^{t2} becomes *S* [*Chord t*1 *V*, *Chord t*2 *I*]

Haskell Implementation: Rules

type *Prob = Double*
type *RuleFun = Dur* $\rightarrow$ *Term*
data *Rule = (CType, Prob)* $:\!\rightarrow$ *RuleFun*

Shorthand functions:
i t = Chord t I   :: RuleFun
ii t = Chord t II :: RuleFun
etc.

$r1 = (I, 0.2) :\!\rightarrow i$

$$I^t \rightarrow I^t$$

$r2 = (I, 0.2) :\!\rightarrow$
$\lambda t \rightarrow S\ [v\ (t/2),\ i\ (t/2)]$

$$I^t \rightarrow V^{t/2}\ \ I^{t/2}$$

$r3 = (V, 0.10) :\!\rightarrow (\ Mod\ M5\ .\ i\ )$

$$V^t \rightarrow M_5(I^t)$$

$r4 = (I, 0.1) :\!\rightarrow \lambda t \rightarrow$
Let “x” (i (t/2)) S [Var “x”, Var “x”]

$I^t \rightarrow$ **let** $x = I^{t/2}$ **in** $x\ x$

Example of Generative Algorithm

<u>Rules Applied (Stochastic)</u>

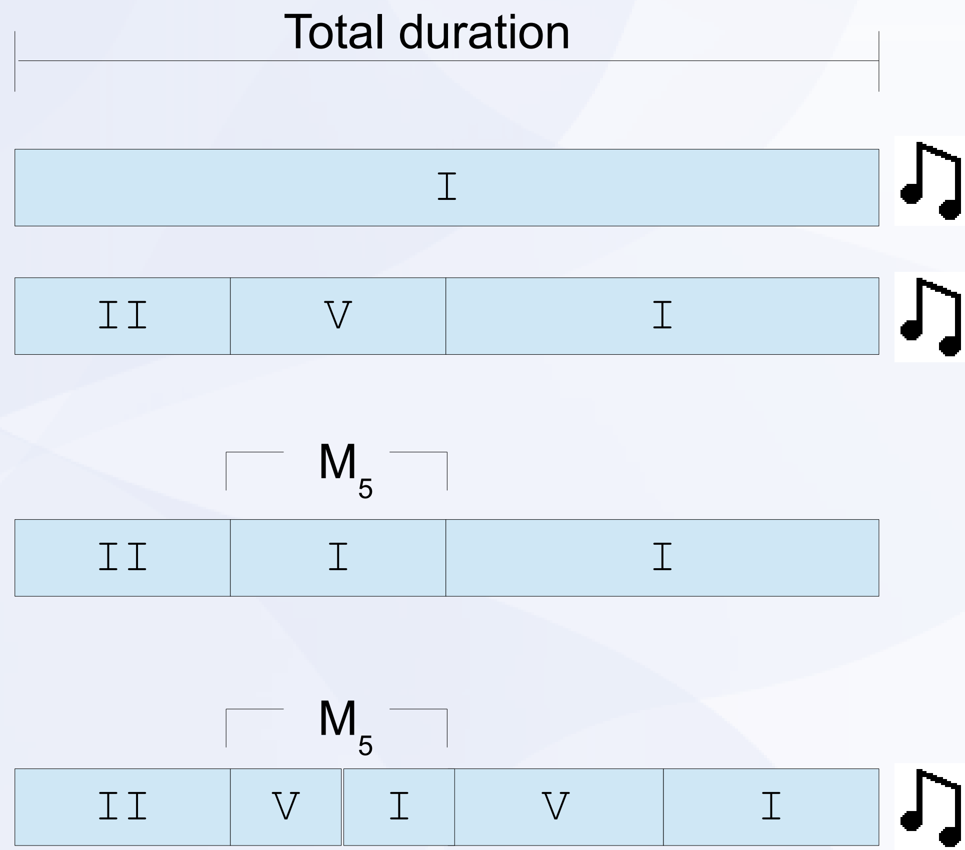
Start symbol: $\mathtt{I}^{t}$

$\mathtt{I}^{t} \rightarrow \mathtt{II}^{t/4}\ \mathtt{V}^{t/4}\ \mathtt{I}^{t/2}$

id, $\mathtt{V}^{t} \rightarrow \mathtt{M}_{5}\ (\mathtt{I}^{t})$, id

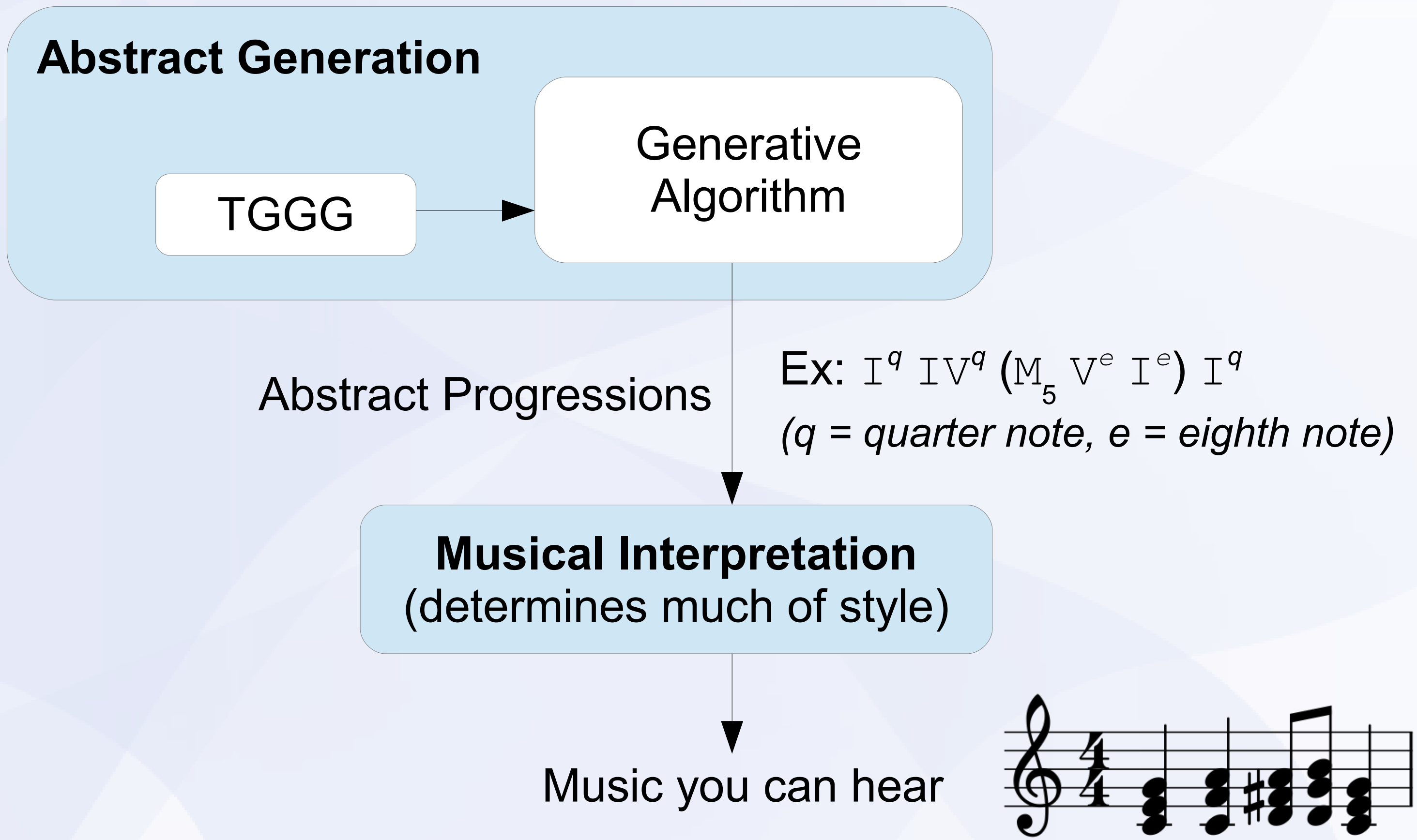
(id means $c^{t} \rightarrow c^{t}$)

id, $\mathtt{I}^{t} \rightarrow \mathtt{V}^{t/2}\ \mathtt{I}^{t/2}$, $\mathtt{I}^{t} \rightarrow \mathtt{V}^{t/2}\ \mathtt{I}^{t/2}$



For **let** *x* = *A* **in** *B*, the phrases *A* and *B* are generated separately, leaving instances of $x \in B$ unaltered. Then, instances of *x* can be instantiated.

Musical Interpretation



We use **chord spaces** as an integral part of our interpretation.

Chord Spaces

- Mathematically grouping chords in musically useful ways.
  - Each chord belongs to an **equivalence class**.
- Examples generated with classical chord spaces [1,2] and also “jazz spaces.”
- Assigning pitches to Roman numerals reduces to a path-finding and **constraint-satisfatction** problem [3].
  - For each **abstract** chord, choose a **concrete** chord from its equivalence class meeting some criteria.
  - *Let* constraints shrink the search space!

[1] C. Callender et al., “Generalized voice-leading spaces,” Science Magazine 2008.
[2] D. Tymoczko et al., “The geometry of musical chords.” Science Magazine, 2006.
[3] D. Quick and P. Hudak, “Computing with chord spaces,” ICMC 2012.

Let Constraints and Chord Spaces

- Progression: **let** x = P Q **in** x x
  ⇒ P Q P Q
- Chord space: P~{a,b}, Q~{c,d}
  Imposed ordering/indices: 0 1 0 1

P & Q are **abstract** chords, like I or V

a, b, c, & d are **concrete** chords

Depth first without lets:

#	Ind.	Value	Satisfies Lets?
1	0000	acac	Yes
2	0001	acad	No
3	0010	acbc	No
4	0011	acbd	No
5	0100	adac	No
...	…	…	…
64	1111	bdbd	Yes

Depth first WITH lets:

#	Ind.	Value	Satisfies Lets?
1	0000	acac	Yes
2	0101	adad	Yes
3	1010	bcbc	Yes
4	1111	bdbd	Yes

Constrained indices move in lock-step, dramatically reducing the number of solutions explored.

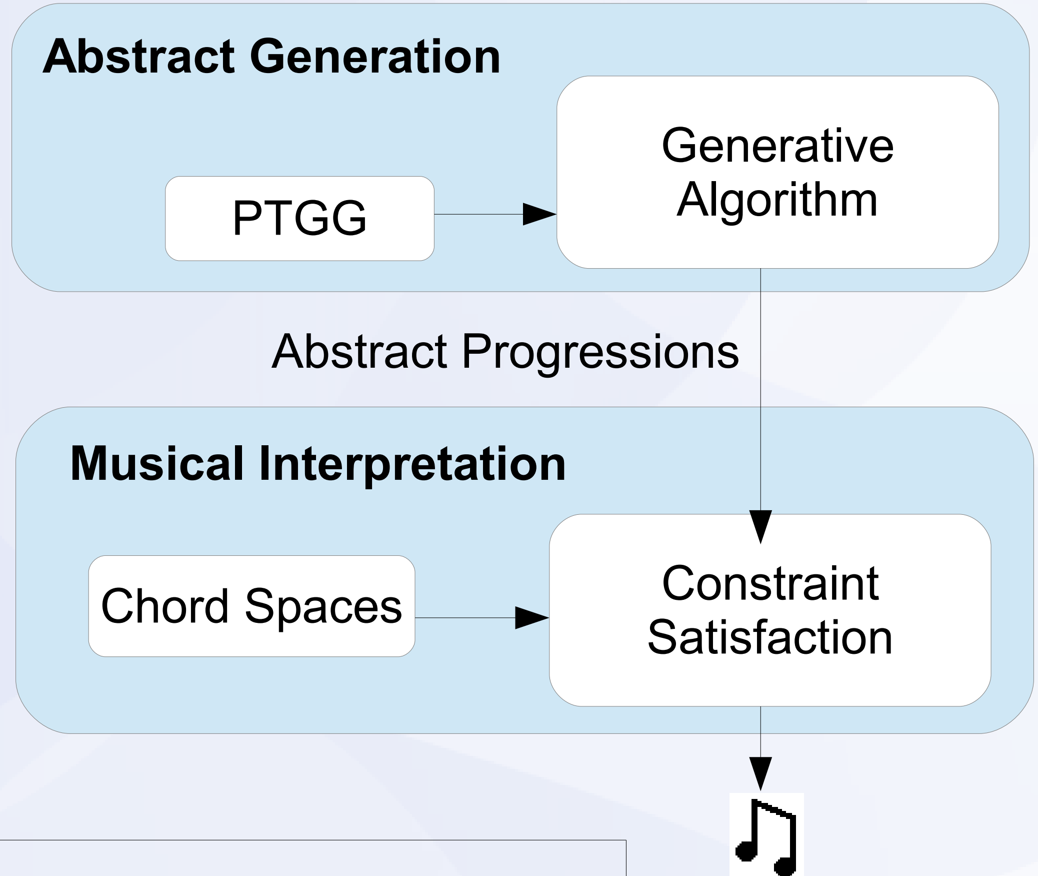
System #1

8-measure example.

Classical chord space for 4 voices.

Shows repetition from nested Let expressions.

NO extra musical post-processing!



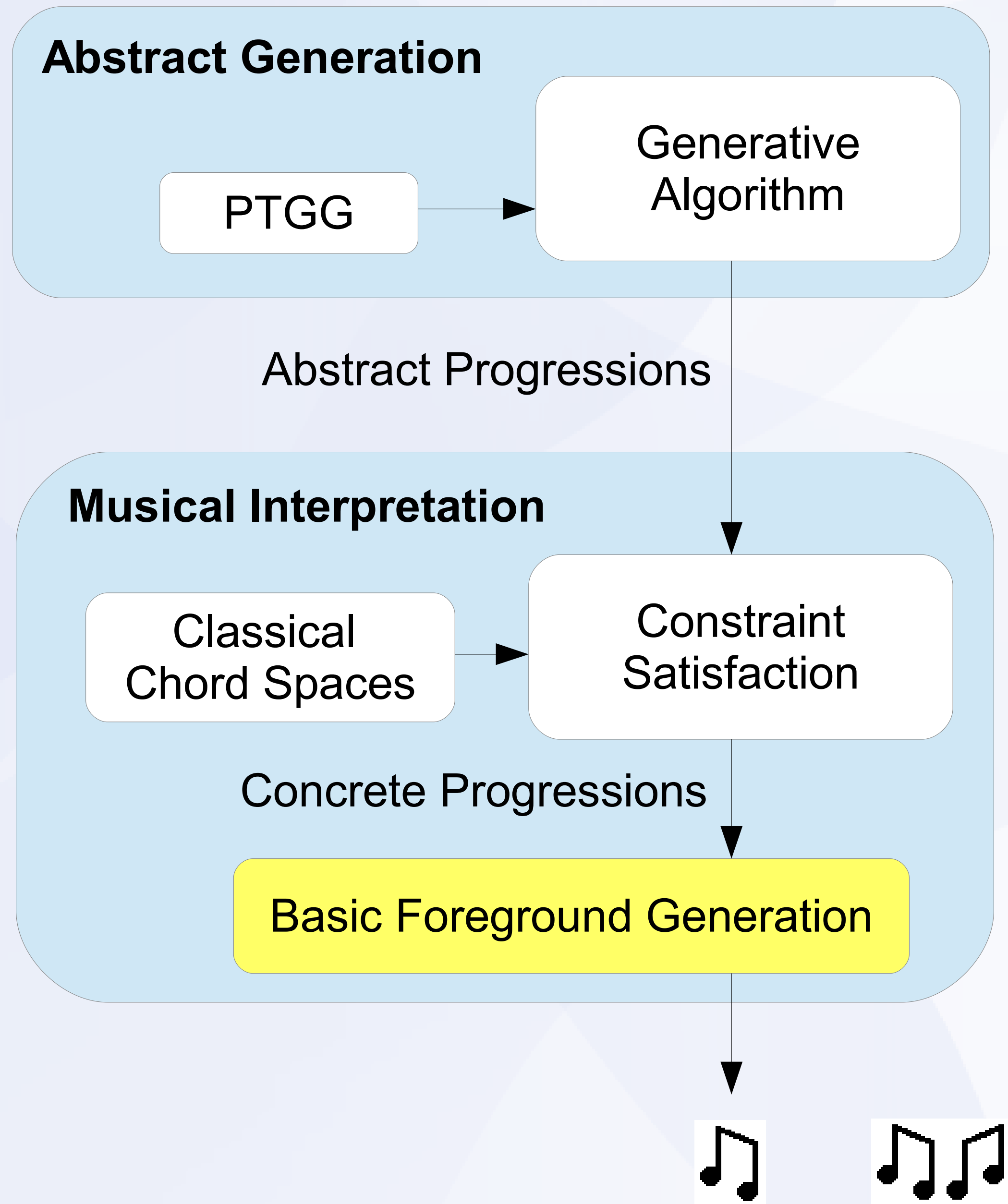
Same System, More Examples

Classical chord spaces

Jazz chord spaces with a syncopated rule set.

System #2

Simple Classical Music



Uses classical chord spaces for 4 voices.

Foreground features added include passing and neighboring tones.

Bach chorale for comparison:

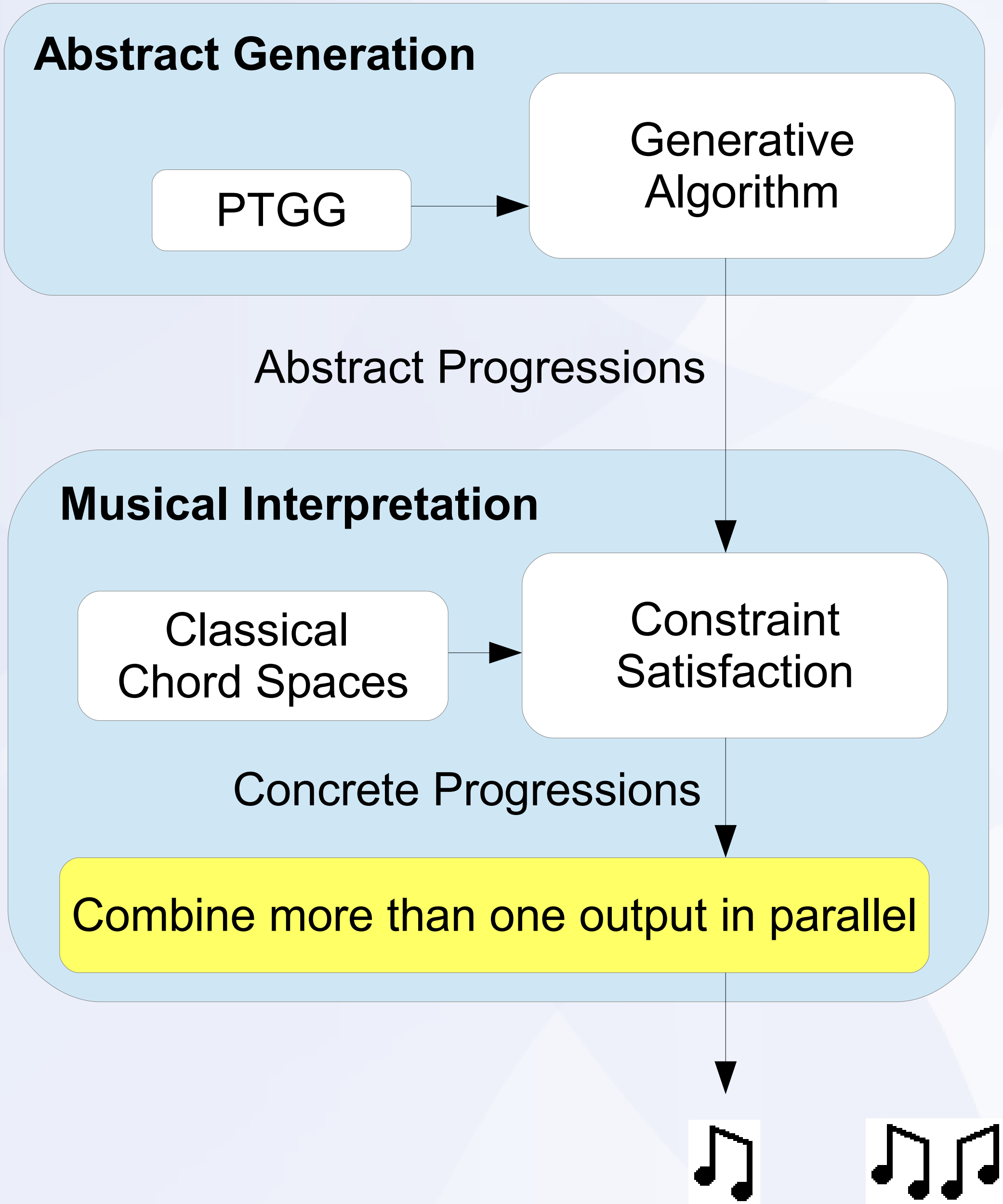
System #3

Modern, texturally interesting music

Uses classical chord spaces for 4 voices.

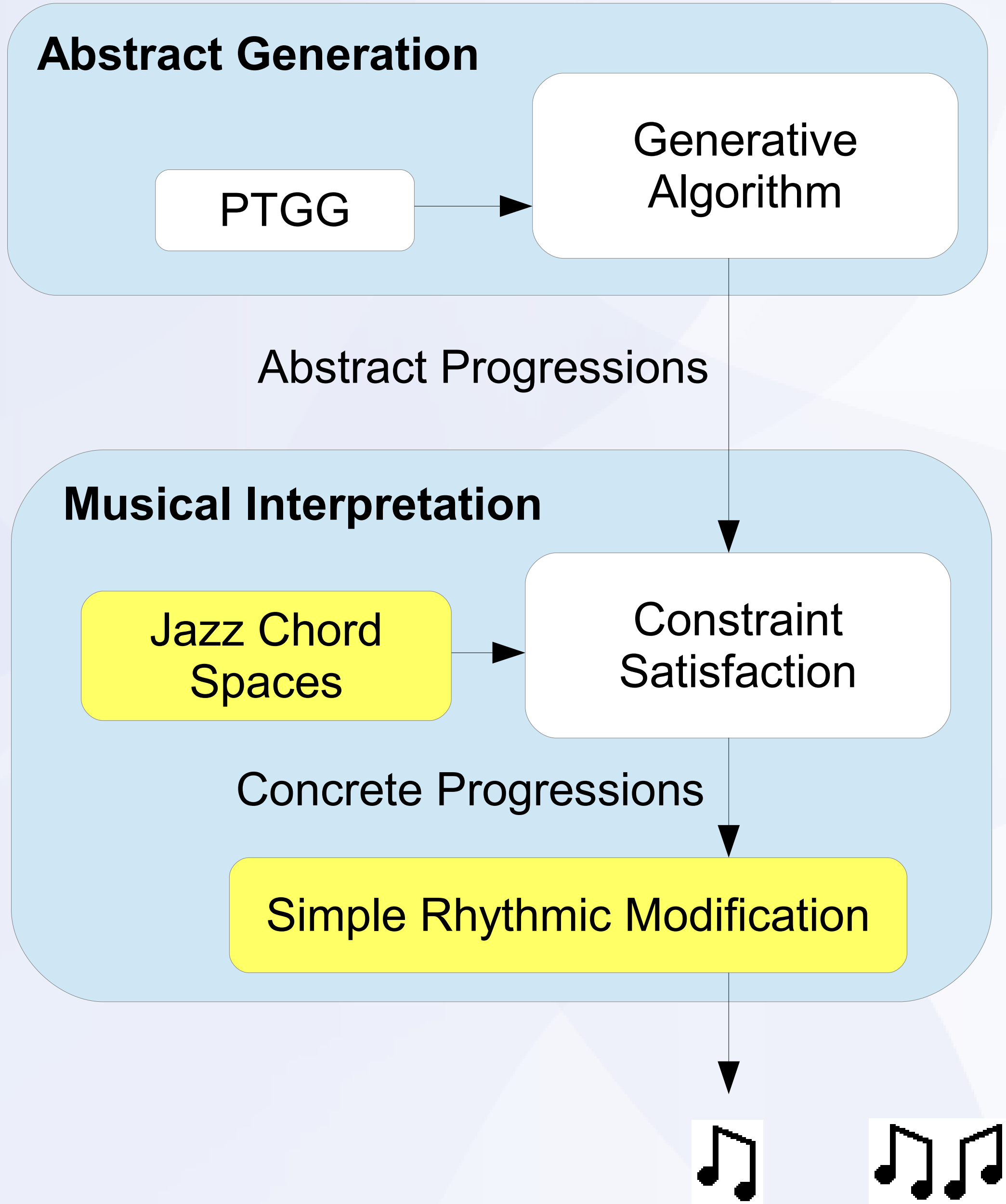
Parts were generated independently and later combined.

Human-controlled: volume changes, staggering of voices, choice of seeds



System #4

Jazz Harmonies



Jazz chord spaces add seconds and sevenths for 5 voices.

Lowest voice's rhythm was stochastically altered.

Conclusions

- A functional approach to modeling music gives us:
  - An elegant Haskell implementation.
  - *Let* expressions that support repetition of phrases.
- Chord spaces allow many different musical styles.
- Areas of potential future work:
  - Melody – currently left to post-processing.
  - More diverse rhythmic support (3/4, triplets in 4/4, etc.)
  - Larger-scale/more complex developmental patterns
    - Theme and variations, partial repetition, etc.
  - Empirical testing with human subjects.
    - How well is a particular style reproduced?

Thank You!

- Implementation at: `haskell.cs.yale.edu`
- Full recordings of examples at: `soundcloud.com/donyaquick`

  Monadic Algorithm Compositions 1, 2, and 3

Complete Rule Set

Num.	Probability	Rule
1	0.20	$I^t \rightarrow II^{t/4}\ V^{t/4}\ I^{t/2}$
2	0.20	$I^t \rightarrow I^{t/4}\ IV^{t/4}\ V^{t/4}\ I^{t/4}$
3	0.20	$I^t \rightarrow V^{t/2}\ I^{t/2}$
4	0.20	$I^t \rightarrow I^{t/4}\ II^{t/4}\ V^{t/4}\ I^{t/4}$
5	0.20	$I^t \rightarrow I^t$
6	0.80	$II^t \rightarrow II^t$
7	0.20	$II^t \rightarrow (M_2\ V^{t/2}\ I^{t/2})$
8	0.70	$III^t \rightarrow III^t$
9	0.30	$III^t \rightarrow (M_3\ I^t)$
10	0.80	$IV^t \rightarrow IV^t$
11	0.20	$IV^t \rightarrow (M_4\ I^{t/4}\ V^{t/4}\ I^{t/2})$
12	0.10	$V^t \rightarrow V^t$
13	0.15	$V^t \rightarrow IV^{t/2}\ V^{t/2}$
14	0.10	$V^t \rightarrow III^{t/2}\ VI^{t/2}$
15	0.10	$V^t \rightarrow I^{t/4}\ III^{t/4}\ VI^{t/4}\ I^{t/4}$

Num.	Probability	Rule
16	0.10	$V^t \rightarrow V^{t/4}\ VI^{t/4}\ VII^{t/4}\ V^{t/4}$
17	0.10	$V^t \rightarrow V^{t/2}\ VI^{t/2}$
18	0.10	$V^t \rightarrow III^t$
19	0.05	$V^t \rightarrow (M_7\ V^t)$
20	0.10	$V^t \rightarrow VII^t$
22	0.10	$V^t \rightarrow (M_5\ I^t)$
22	0.70	$VI^t \rightarrow VI^t$
23	0.30	$VI^t \rightarrow (M_6\ I^t)$
24	0.40	$VII^t \rightarrow VII^t$
25	0.50	$VII^t \rightarrow I^{t/2}\ III^{t/2}$
26	0.10	$VII^t \rightarrow (M_7\ I^t)$

Extra Let rules for all $c \in C$:

$c^t \rightarrow$ let x = $c^{t/2}$ in x x

$c^t \rightarrow$ let x = $c^{t/4}$ in x $c^{t/2}$ x

$c^t \rightarrow$ let x = $c^{t/4}$ in x $\mathbb{V}^{t/2}$ x

Voice-Leading with Chord Spaces

Input chords (abstract):
Cmaj, Gmaj, …